UNITED STATES NAVAL

TEST PILOT SCHOOL

AIRBORNE SYSTEMS COURSE

TEXTBOOK

INTEGRATED WEAPON SYSTEM

TEST AND EVALUATION

By

George W. Masters

26 October, 1981

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Textbook for teaching test and evaluation of integrated weapon systems, including system theory of operation, operating characteristics, and test methodology. Topics include weapon delivery systems, integrated weapon systems, the philosophy of testing, test planning, test program management, data acquisition and processing and data analysis.

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INTEGRATED WEAPON SYSTEM

TEST AND EVALUATION

TABLE OF CONTENTS

1.0   Introduction

1.1 The Nature of Integrated WeaponsSystems

A modern airborne weapon system is, in fact, a system of systems. The com-
ponent sub-systems are integrated into a "super system" with capabilities
beyond the combined capabilities of the sub-systems operating independently.
For that reason, the evaluation of airborne weapons system performance requires
tests concerned not only with the performance of the sub-systems, but also with
system integration -- that is, how well the various sub-systems perform their
integrated function. In that context, the entire aircraft can be considered a
weapons system.

The essential functions of an integrated airborne weaponssystem are listed below.

        Target Detection, Location, and Identification
        Target Tracking
        Target Interception
        Weapon "Aiming"
        Weapon Release
        Post-Release Weapon Control
        Post-Release Aircraft Maneuvering
        Target Destruction

The block diagram of an integrated airborne weapons system is shown in Figure
1.1.0.1. The sub-systems involved are listed below and discussed in the following
paragraphs.

        Weapons System Computer
        Multiplex Data Bus
        Communications System

Controls and Displays
Navigation System
Air Data System
Target Sensor Systems
Flight Control System
Weapons
Electronic Warfare System

Weapons System Computer -- The weapons system computer is the primary integrating mechanism for the weapons system. It receives and processes inputs from the various sub-systems, determines the proper course of action, and transmits appropriate commands and signals to the sub-systems involved in the required operation. In modern aircraft, the weapons system computer is generally digital. (See Section 2.2 of the communications text for a discussion of the characteristics of digital systems.) The block diagram of Figure 1.1.0.1 indicates only the computer that performs the integrating function. In some weapons systems, the integrating computer also performs computational functions for the individual sub-systems. In other weapons systems, the sub-systems incorporate separate computers dedicated to their tasks. From the viewpoint of computational efficiency, centralization of the computating facilities in a weapons system is desirable. When a centralized computer is employed, however, the resulting interaction of the computations associated with the various sub-systems greatly complicates the task of system integration testing.

Multiplex Data Bus -- In a modern airborne weapons system, the various sub-systems may communicate by means of a multiplex data bus. A multiplex data bus simultaneously carries all information between the sub-systems by frequency - or time-division multiplexing. For purposes of performance evaluation, the existence of a multiplex data bus greatly facilitates access to the required information.

Controls & Displays

Flight Control
System

Weapons

Electronic Warfare
System

Multiplex
Data Bus

Communications
System

Navigation
System

Air Data
System

Target Sensor
Systems

Weapons System
Computer
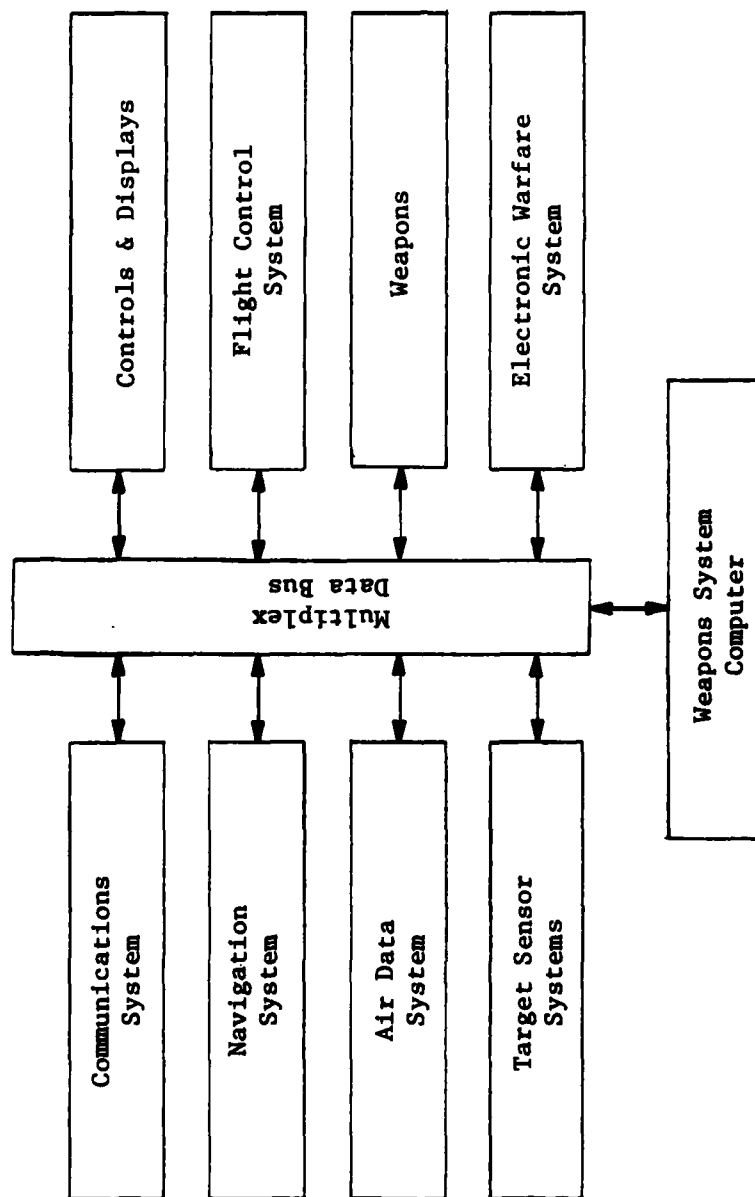
Figure 1.1.0.1--Integrated Airborne Weapon System

1.1a

(When a multiplex data bus is not employed, adequate monitoring of a digital system requires that the system designer provide for access to internal (not-normally externally available) signals.

Communications System -- The communications system provides both voice communication and data link with the outside world. Two-way voice communication and data links are generally provided, thus allowing systems external to the aircraft to become part of the overall weapons system. For a detailed discussion of the characteristics of communications systems, the reader is referred to the text on that subject. In weapon system evaluation, external communication and data links must be included in the test plan.

Controls and Displays -- The controls and displays provide the interface between the crew and the various sub-systems, thus making the crew an integral part of the weapons system. Human factors evaluation is an important aspect of weapons system testing.

Navigation System -- The navigation system provides information for both target interception and weapon delivery. The latter category includes vehicle position, velocity, and attitude and, indirectly, wind velocity.

Air Data System -- The air data system provides information as to the altitude, airspeed, angle of attack, and angle of sideslip of the aircraft. This information is used in weapon delivery and in back-up-mode navigation.

Target Sensor Systems -- The primary target sensors are radar and electro-optical systems. For a detailed discussion of these sensor systems, the reader is referred to the texts on those respective subjects. The function of the target sensors is to determine the position and velocity of the target with respect to the sensing aircraft. For purposes of weapon delivery, these quantities also are transformed into the coordinate system in which the weapon delivery computations are performed.

Flight Control System -- The flight control system provides the interface between the aircraft and the other weapon delivery sub-systems. (The aircraft is the one indispensable component of an airborne weapon delivery system.) In the delivery of weapons without post-launch guidance, accuracy is ultimately determined by aircraft control. In some systems, the role of the flight control system is to stabilize the flight path of the weapons platform (aircraft). In fully automated systems, the flight control system also executes the commands of the weapon system computer.

Weapons -- It is the function of the airborne weapons to effect final intercept and destruction of the target. Airborne weapons are discussed in detail in Section 2.0 of this text.

Electronic Warfare System -- While the detection and jamming of hostile sensors is not fundamental to weapon delivery, electronic warfare is an integral and essential part of a weapons system. In particular, electronic counter-counter-measures are essential to successful weapon delivery. For a detailed discussion of electronic warfare, the reader is referred to the text on that subject.

2.0 Airborne Weapons Systems Theory of Operation

2.1 Types of Airborne Weapons

Airborne weapons fall into two major categories -- those with post-launch guidance and those without post-launch guidance. The unguided weapons include bombs, guns, and rockets. The guided weapons include bombs, rockets, torpedoes, and mines.

2.2 The Weapon Delivery Problem

In general terms, the weapon delivery problem entails the following processes.

(1) Determine the present and predicted future position of the target

(2) Predict the post-launch trajectory of the missile (bomb, bullet, or rocket).

(3) Release the missile in such a way that its trajectory intercepts that of the target.

(4) For guided missiles, control the trajectory of the missile so as to intercept the target.

In order to predict the post-launch trajectory of the missile, the missile differential equations of motion must be integrated. The generalized equations of motion of an airborne missile are presented below. (Airborne weapon delivery computations typically employ a greatly simplified version of these equations.)

$$\dot{\mu} = rv - g\omega - \left(\frac{\rho A}{2m}\right) V_T^2\, c_x + Fx/m$$

$$\dot{v} = -r\mu + pw - \left(\frac{\rho A}{2m}\right) V_T^2\, c_{y\beta}\, \beta + Fy/m$$

$$\dot{\omega} = g\mu - pv - \left(\frac{\rho A}{2m}\right) V_T^2\, c_{z\alpha}\, \alpha + Fz/m$$

$$\dot{p} = \left(\frac{\rho A d^2}{2 I_x}\right) V_T\, c_{\ell p}\, p + \left(\frac{\rho A d}{2 I_x}\right) V_T^2\, c_{\ell_o} + Mx/I_x$$

$$\dot{q} = \left(\frac{\rho A d^2}{2 I_y}\right) V_T\, c_{mq}\, q + \left(\frac{\rho A d}{2 I_y}\right) V_T^2\, c_{m\alpha}\, (\alpha - \alpha_T)$$
$$\quad + \left(\frac{I_z - I_x}{I_y}\right) pr + My/I_y$$

$$\dot{r} = \left(\frac{\rho A d^2}{2 I_z}\right) V_T\, c_{nr}\, r - \left(\frac{\rho A d}{2 I_z}\right) V_T^2\, c_{n\beta}\, (\beta - \beta_T)$$
$$\quad - \left(\frac{I_y - I_x}{I_z}\right) pq + Mz/I_z$$

2.2

where:

$$\alpha = Tan^{-1}\left[\omega/\mu\right]$$

$$\beta = Sin^{-1}\left[v/v_T\right]$$

$$V_T = \left[\mu^2 + v^2 + \omega^2\right]^{1/2}$$

$$\mu = \dot{x}$$
$$v = \dot{q}$$
$$\omega = \dot{z}$$

The quantities appearing in these equations are defined below and in Figure 2.2.0.1.

A = Reference Area

c_{ℓ}, c_m, c_n = Aerodynamic Moment Coefficients

c_x, c_y, c_z = Aerodynamic Force Coefficients

c_{xx} = Aerodynamic Coefficient Partial Derivatives

d = Reference Distance

F_x, F_y, F_z = Non-Aerodynamic Forces

I_x, I_y, I_z = Moments of Inertia

m = Mass

M_x, M_y, M_z = Non-Aerodynamic Moments

p, q, r = Angular Velocities

μ, v, ω = Linear Velocities

V_T = Velocity with respect to Air Mass

x, y, z = Linear Displacements in Body-Fixed Inertial Coordinates

α = Angle of Attack

α_T = Trim Angle of Attack

β = Angle of Side Slip

β_T = Trim Angle of Side Slip

ρ = Air Density (Function of Altitude)

2.3

x,u

p

y,v

q

z,w

r

cg
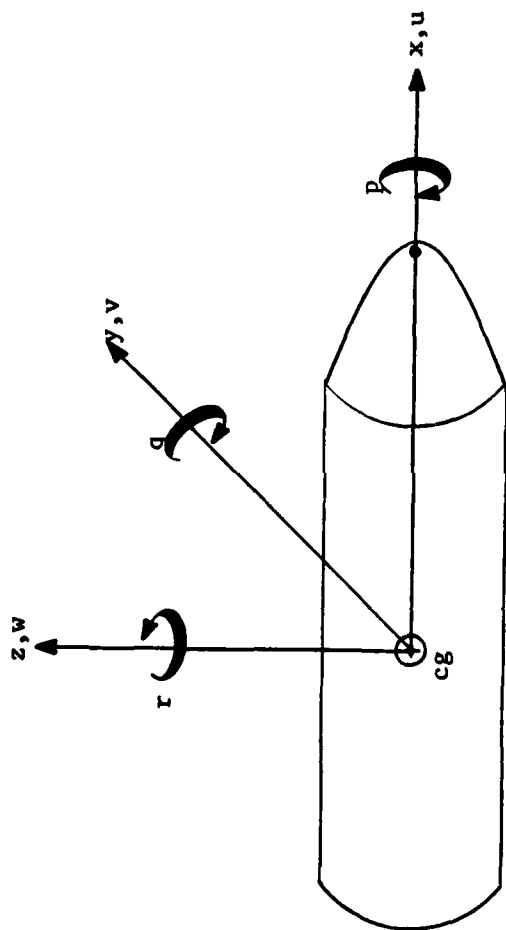
cg = Center of Gravity
p = Roll Rate
q = Pitch Rate
r = Yaw Rate

u,v,w = Linear Velocities
x,y,z = Linear Displacements

Figure 2.2.0.1--Missile Coordinate System

2.3a

In addition to integrating the equations of motion, the weapon delivery computations require transformation from a coordinate system fixed to the missile body to a coordinate system fixed to the air mass. Defining that transformation by means of the Euler angles illustrated in Figure 2.2.0.2, the transformation is as follows.

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = [M] \begin{bmatrix} \mu \\ \nu \\ \omega \end{bmatrix}$$

where the transformation matrix, M, is given by:

$$[M] = \begin{bmatrix} c\vartheta\, c\psi & s\phi\, s\vartheta\, c\psi - c\phi\, s\psi & c\phi\, s\vartheta\, c\psi + s\phi\, s\psi \\ c\vartheta\, s\psi & s\phi\, s\vartheta\, s\psi + c\phi\, c\psi & c\phi\, s\vartheta\, s\psi - s\phi\, c\psi \\ -s\vartheta & s\phi\, c\vartheta & c\phi\, c\vartheta \end{bmatrix}$$

where:

$\quad$ Cθ = Cosine of θ

$\quad$ Sθ = Sine of θ

$\quad$ μ, ν, ω = Linear Velocities in Body-Fixed Coordinates

$\quad$ $\dot{X}$, $\dot{Y}$, $\dot{Z}$ = Linear Velocities in Air Mass-Fixed Coordinates
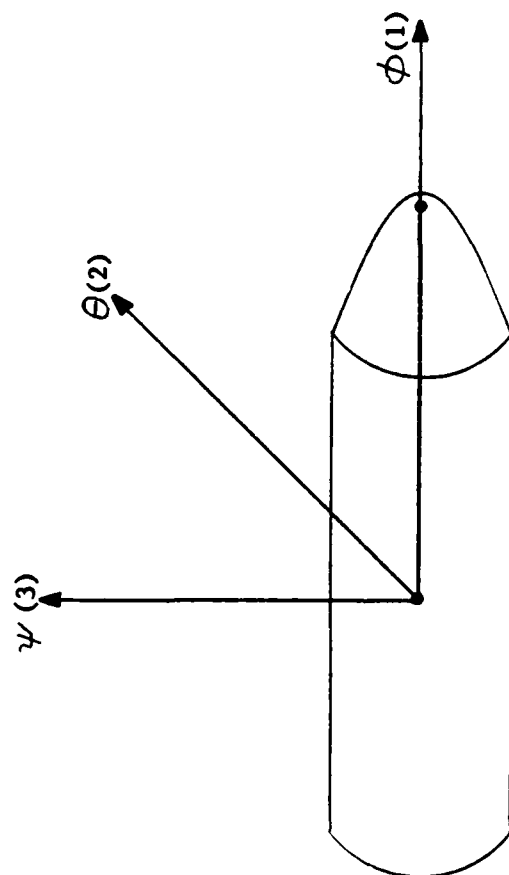
Figure 2.2.0.2--Euler Angles for Body-Fixed to Air Mass-Fixed Coordinate Transformation

2.4a

Another mathematical operation required in weapon delivery computations is the transformation from the air-mass-fixed coordinate system to an earth-fixed coordinate system. Assuming steady, non-rotational motion of the air mass with respect to the earth, the transformation consists of a simple vector addition of the wind velocity to the missile velocity with respect to the air mass. That is:

$$\dot{X}_G = \dot{X} + V_{\omega x}$$

$$\dot{Y}_G = \dot{Y} + V_{\omega y}$$

$$\dot{Z}_G = \dot{Z} + V_{\omega z}$$

where:

$V_{\omega x}$, $V_{\omega y}$, $V_{\omega z}$ = x, y, and z components of wind velocity with respect to the earth

$\dot{X}$, $\dot{Y}$, $\dot{Z}$ = x, y, and z components of missile velocity with respect to the air mass.

$\dot{X}_G$, $\dot{Y}_G$, $\dot{Z}_G$ = x, y, and z components of missile velocity with respect to the earth.

In general, the wind velocity is a function of altitude and the vertical component of wind is assumed to be zero.

The weapon delivery computations entail the integration of the missile differential equations of motion with coordinate transformations as required. The computed trajectory of the missile is then compared with the predicted trajectory of the target. When the computed position of the missile coincides with the predicted position of the target at the time of impact, the missile is launched.

The integration of the missile equations of motion can be performed in real time or they can be performed in advance and stored. If stored weapon delivery solutions are used, the actual launch conditions must be constrained to those for which the stored solutions were computed. The requirement for duplication of launch conditions introduces potential weapon delivery errors. In addition, it may be impossible or highly undesirable (due to hostile action) to duplicate a given set of launch conditions. For these reasons, the real-time, iterative integration (solution) of the weapon delivery equations is generally preferred.

Weapon Impact in Ground Plane

E_d

E_r

Target
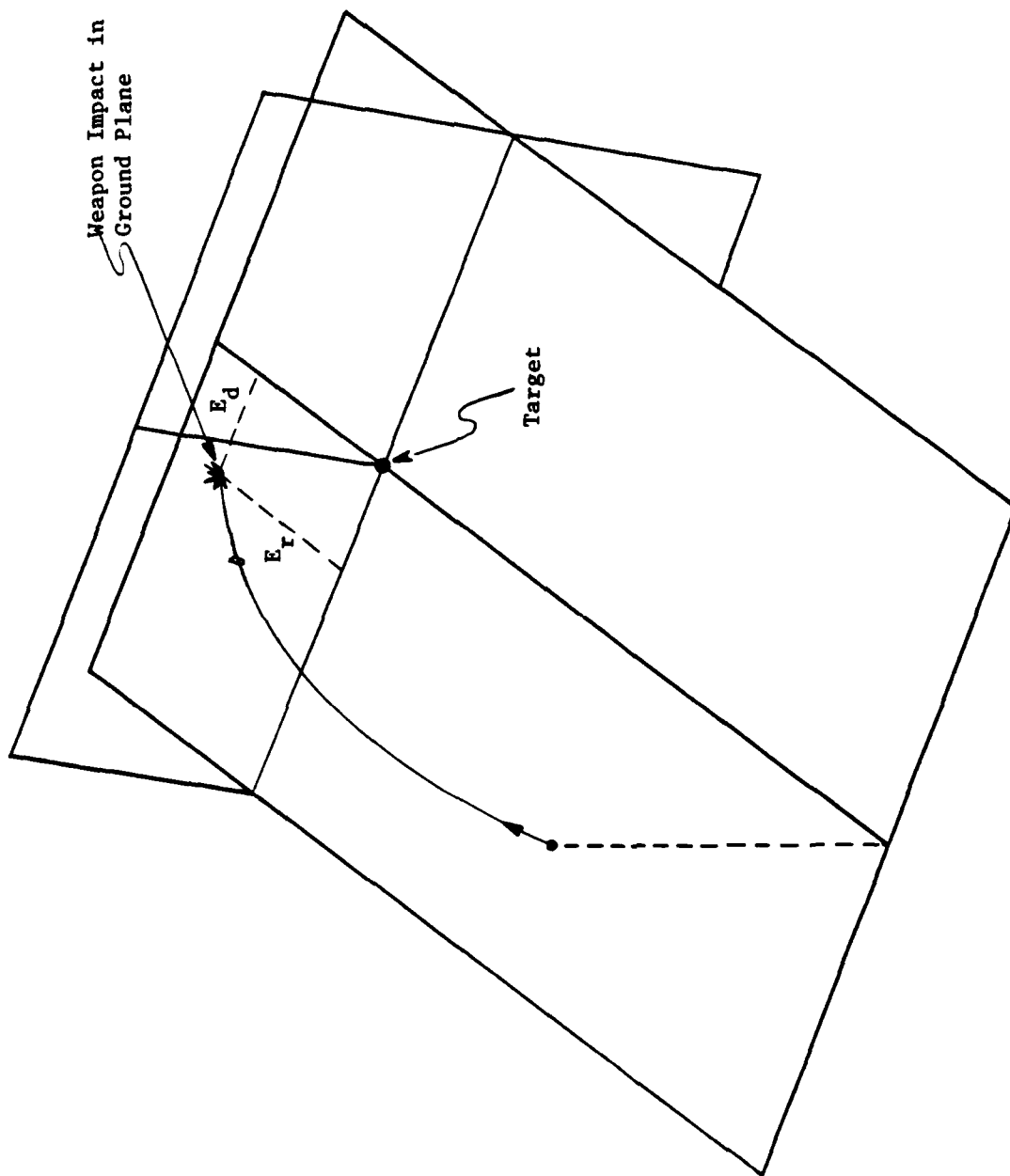
E_d = Linear Miss Distance in Deflection (Cross-Range) (Meters)

E_r = Linear Miss Distance in Range (Meters)

Figure 4.2.1.2 -- Air-To-Ground Weapon Scoring in Ground Plane

4.6a

2.3 Unguided Bomb Delivery

The factors affecting the trajectory of an unguided bomb are listed below.

> Release Position
> Release Velocity with respect to Air Mass
> Air Mass Velocity with respect to Earth
> Release Attitude
> Bomb Aerodynamic Characteristics
> Bomb Mass

The differential equations of motion presented in Section 2.2 of this text are simplified, for unguided bomb delivery, by the following assumptions:

(1) No Normal Forces ($\alpha = \beta = o$)

(2) Negligable Gyroscopic Forces ($p = q = r \doteq o$)

(3) Motion with respect to Air Mass in Vertical Plane ($y = \dot{y} = o$)

(4) Non-Aerodynamic Forces due only to Gravity

With these simplifying assumptions, the unguided bomb differential equations of motion reduce to the following form (refer to Figure 2.3.0.1.):

$$\ddot{x} = -\left(\frac{\rho K_D}{2m}\right) V_T^2 \cos(\gamma)$$

$$\ddot{z} = -\left(\frac{\rho K_D}{2m}\right) V_T^2 \sin(\gamma) - g$$

or:

$$\ddot{x} = -\left(\frac{\rho K_D}{2m}\right)(\dot{x})\left[(\dot{x})^2 + (\dot{z})^2\right]^{1/2}$$

$$\ddot{z} = -\left(\frac{\rho K_D}{2m}\right)(\dot{z})\left[(\dot{x})^2 + (\dot{z})^2\right]^{1/2} - g$$

where:

g = Acceleration due to Gravity

K_D = Bomb Drag Coefficient

V_T = Velocity (True Airspeed)

γ = Flight Path Angle

$$F_D = -\left(\frac{\rho K_D}{2m}\right) v_T^{\,2}$$

x

z

x_O

z_O

v_O

v_T

mg

$\dot{x}$

$\dot{z}$

γ
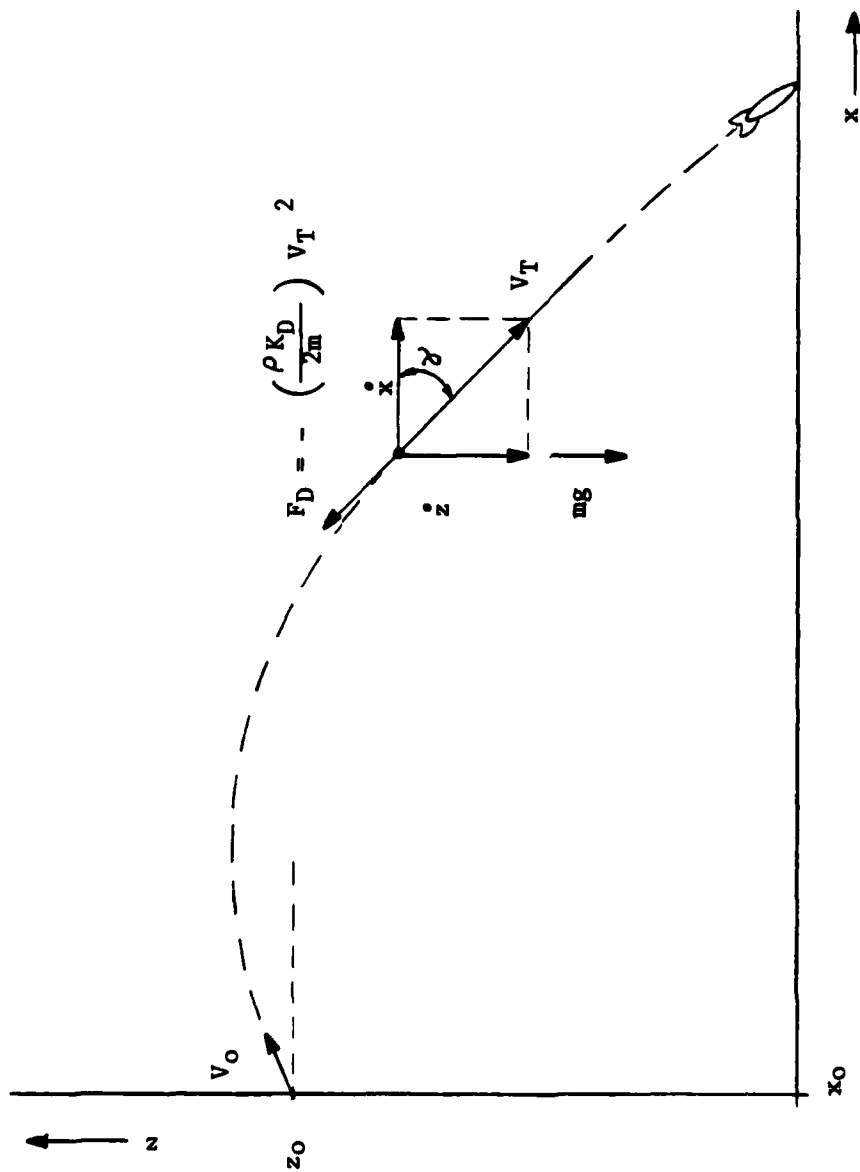
Figure 2.3.0.1--Vertical Plane Bomb Trajectory

2.7a

In these equations, the air density, ρ, is a function of altitude, and the
bomb drag coefficient, K_D, is a function of mach number (altitude and airspeed).
Typical curves of drag coefficient versus mach number are shown in Figure
2.3.0.2 for low-drag and high-drag bombs.

In order to compute the bomb velocity with respect to the air mass, the above
equations are integrated with appropriate initial conditions. The velocity
of the bomb with respect to the earth is then obtained by vectorially adding
the velocity of the air mass with respect to the earth (wind velocity) to that
of the bomb with respect to the air mass. The computed trajectory of the bomb
is then obtained by integrating the velocity of the bomb with respect to the
earth. When the weapon delivery equations are integrated in real time, the
above process is repeated until the computed bomb position coincides with that
of the target, at the time of impact. Typical vertical-and horizontal-plane
bomb trajectories are shown in Figures 2.3.0.3 and 2.3.0.4, respectively.

High – Drag Configuration

Low – Drag Configuration
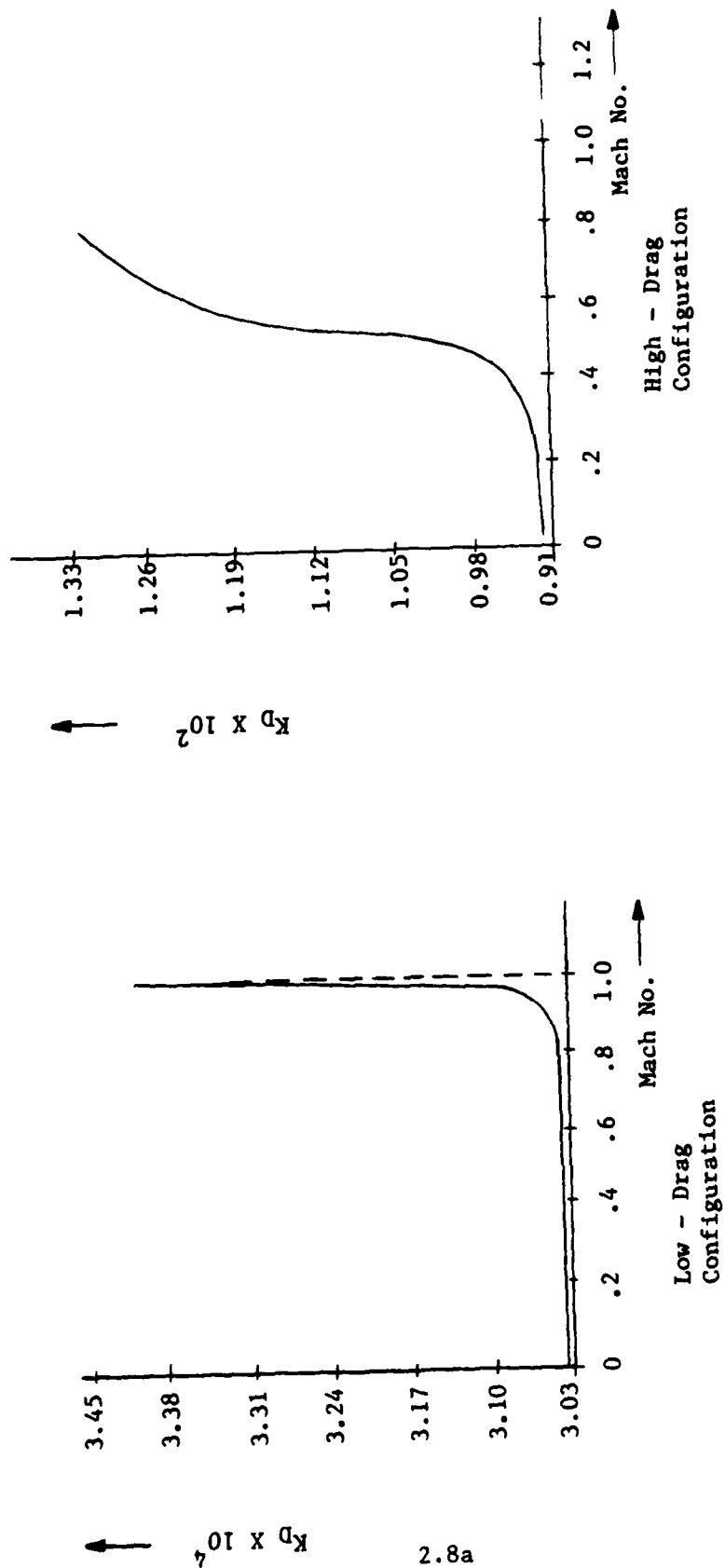
Figure 2.3.6.2--Mark 81 Bomb Drag Coefficients

2.8a

500

400

300

200

100

0

z (Ft)

0   1   2   3   4   5

x (KFt)

High-Drag Bomb

Low-Drag Bomb
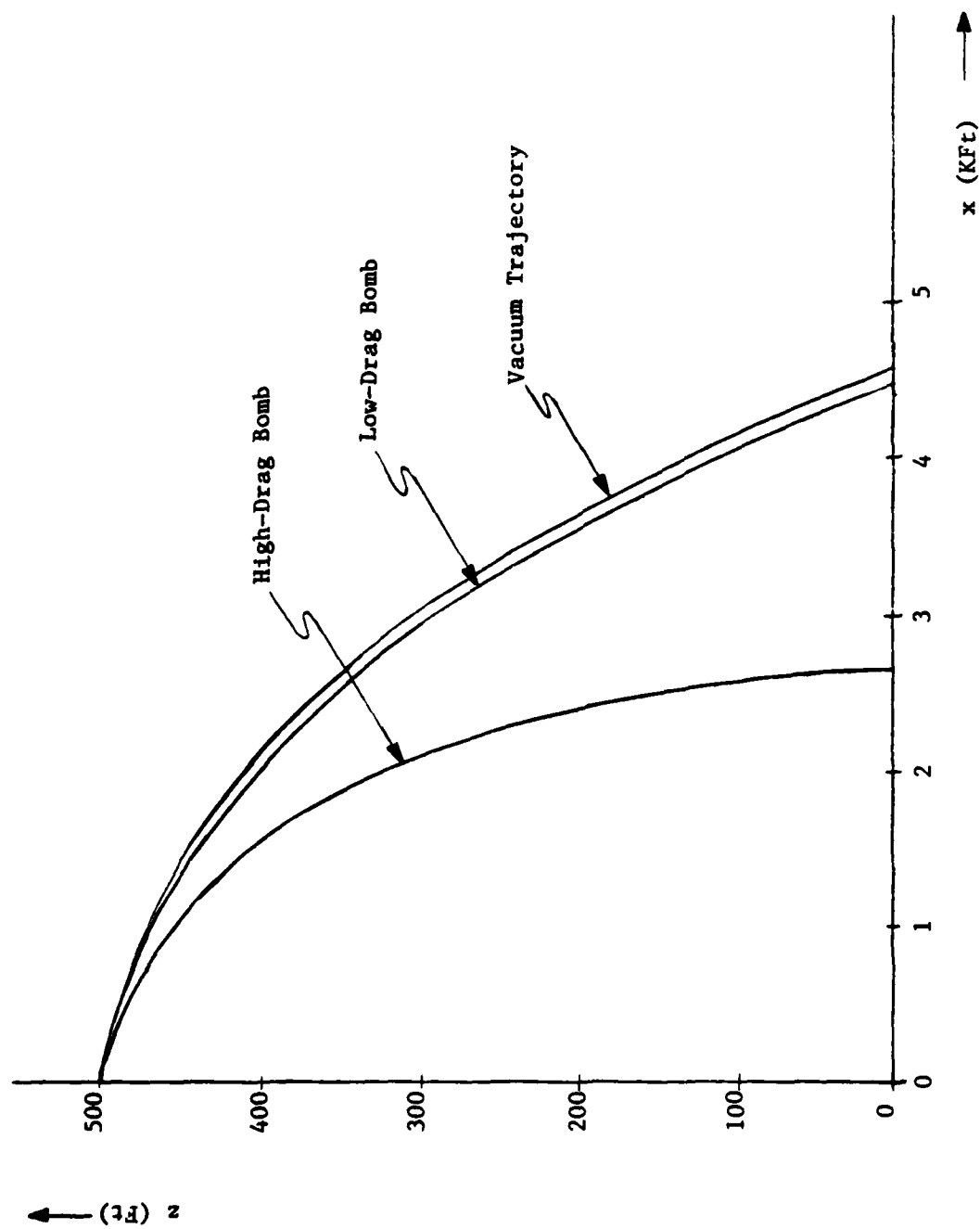
Vacuum Trajectory

Figure 2.3.0.3--Vertical Plane Trajectory of Unguided Bomb

2.8b

Trajectory with wind

Trajectory without wind

V_w

P_3

$V_w t_f$

P_2 P_1

x

V_o

y_o

x_o

y
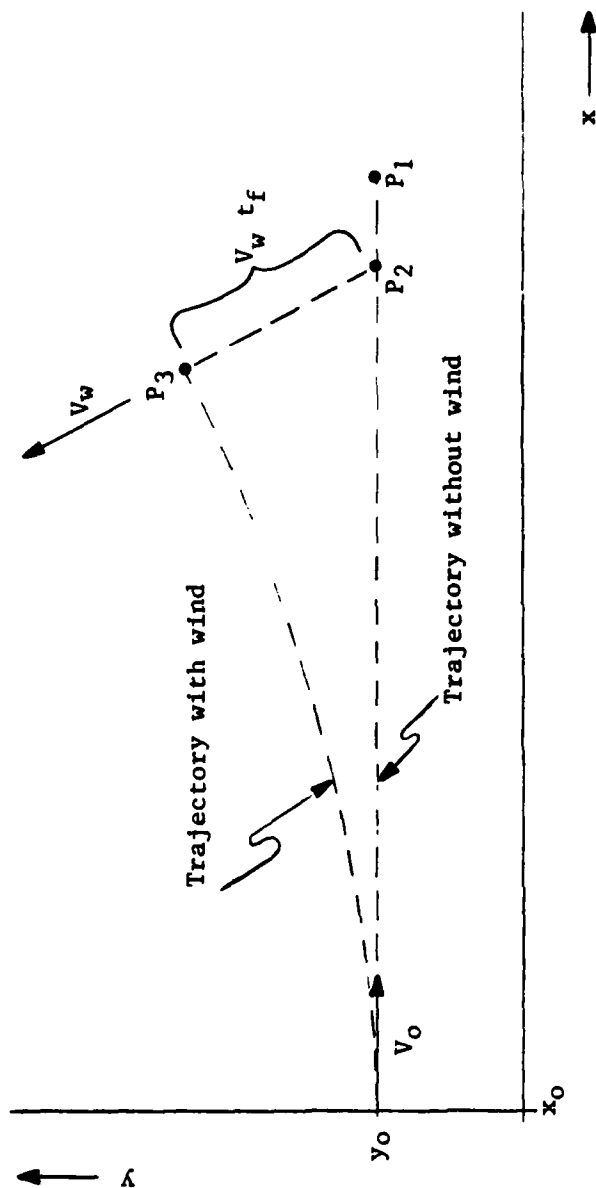
P₁ = Impact Point in Vacuum
P₂ = Impact Point Without Wind
P₃ = Impact Point With Wind
t_f = Time of Flight of Bomb
V_o = Release Velocity of Bomb
V_w = Wind Velocity
x = Range Displacement
y = Cross-Range Displacement

Figure 2.3.0.4--Horizontal-Plane Trajectory of Unguided Bomb

2.8c

2.4 Aerial Gunnery

The factors affecting the trajectory of a gun projectile, listed below, are
identical to those for a bomb.

> Release Position
> Release Velocity with respect to Air Mass
> Air Mass Velocity with respect to Earth
> Release Attitude
> Bullet Aerodynamic Characteristics
> Bullet Mass

The differential equations of motion for a projectile also are identical to
those for a bomb. That is:

$$\ddot{x} = - \left(\frac{\rho K_D}{2m}\right) (\dot{x}) \left[(\dot{x})^2 + (\dot{z})^2\right]^{1/2}$$

$$\ddot{z} = - \left(\frac{\rho K_D}{2m}\right) (\dot{z}) \left[(\dot{x})^2 + (\dot{z})^2\right]^{1/2} - g$$

Despite the identity of their equations of motion, there are significant
differences between the trajectories of bullets and those of bombs. Those
differences are entirely the result of the much greater release velocities of
a bullet. The large release velocity of a bullet affects the equations of
motion in two major ways - - the much shorter time of flight and the much
different drag profile. The result is a flatter trajectory and one in which
the dependence of the release velocity on the velocity of the aircraft is
greatly reduced. Fundamentally, however, the weapon delivery problems for
bombing and air-to-ground gunnery are the same. A typical curve of drag co-
efficient versus mach number is shown in Figure 2.4.0.1 for a Mark 11 20
millimeter projectile.

The gyroscopic forces on a spinning projectile are not negligible and produce
a nutation that significantly increases the drag and also causes the projectile
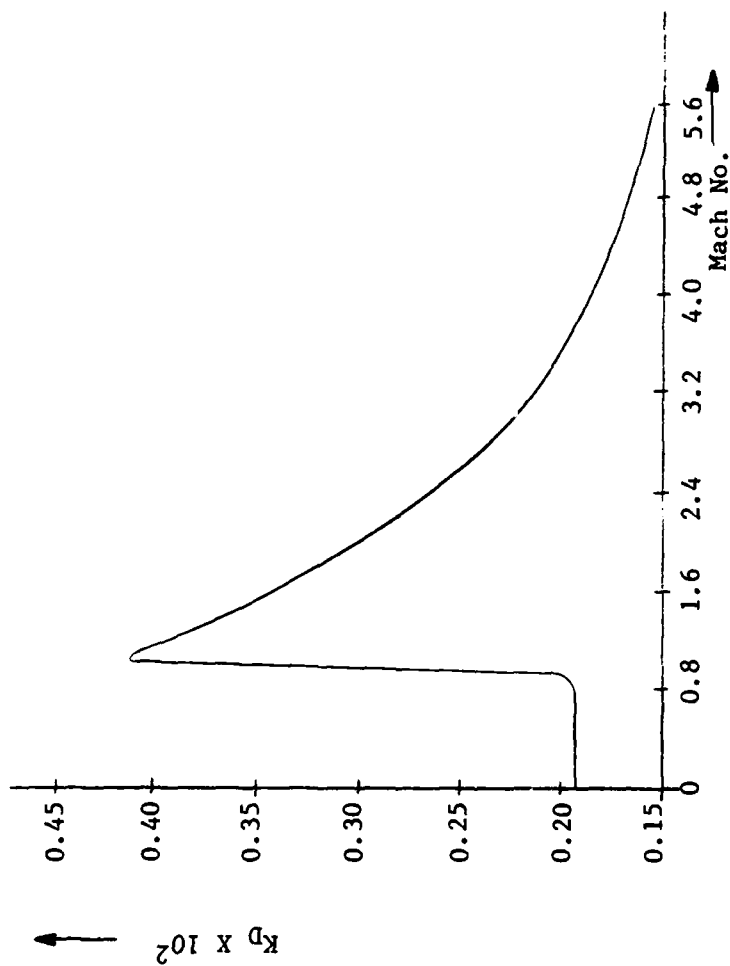
Figure 2.4.0.1--Drag Coefficient Versus Mach Number for a Mark 11 20 mm Projectile

2.9a

to move in a helical path. These forces, and the resulting nutation are not included in the mathematical model (differential equations of motion), however, because of the impossibility of accounting for the phasing of the nutation. The effects of the nutation are included in the dispersions applied to the nominal trajectory of a spinning projectile.

As for bomb delivery, the differential equations of motion for gunnery are integrated to obtain the velocity of the projectile with respect to the air mass. For air-to-ground gunnery, the wind velocity is then added to the computed velocity to obtain the velocity of the projectile with respect to the ground. A second integration then yields the projectile trajectory for comparison with the predicted position of the target. For air-to-air gunnery, the air mass-to-ground transformation is unnecessary. (Air-to-air weapon delivery is computed in air mass coordinates in which there is no "wind" velocity.)

In order to predict the future position (trajectory) of a moving target, the velocity of the target with respect to the air mass must be determined. The target velocity with respect to the air mass can be computed indirectly by measuring the velocity of the target with respect to the attacking aircraft and then adding the velocity of the attacking aircraft with respect to the air mass. The velocity of the target with respect to the aircraft is computed as the vector sum of the radial and tangential velocity components of the target with respect to the aircraft. The radial target velocity is the range rate and the tangential velocity is equal to the product of range and line-of-sight slew rate, quantities normally available from a tracking sensor. The target velocity equations are presented below and the vector geometry is shown in Figure 2.4.0.2.

2.10
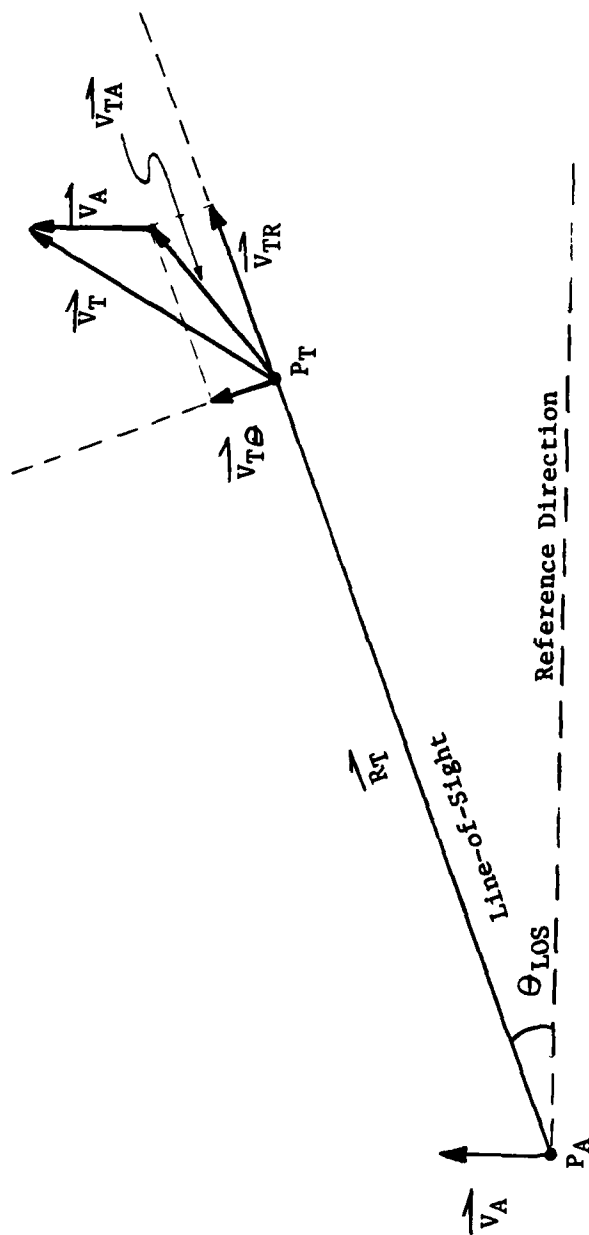
P_A = Position of Attacking Aircraft,
P_T = Position of Target
(See text for definitions of other symbols)

Figure 2.4.0.2--Determination of Target Velocity with respect to Air Mass

2.10a

$$\vec{V_T} = \vec{V}_{TA} + \vec{V}_A$$

$$\vec{V}_{TA} = \vec{V}_{TR} + \vec{V}_{T\theta}$$

$$\vec{V}_{TR} = \dot{\vec{R}}_T$$

$$\vec{V}_{T\theta} = \dot{\vec{\theta}}_{LOS} \times \vec{R}_T$$

where:

$\vec{R}_T$ = Range Vector from Aircraft to Target

$\vec{V}_A$ = Velocity Vector of Aircraft with respect to Air Mass

$\vec{V}_T$ = Velocity Vector of Target with respect to Air Mass

$\vec{V}_{TA}$ = Velocity Vector of Target with respect to Aircraft

$\vec{V}_{TR}$ = Radial Component of $\vec{V}_{TA}$

$\vec{V}_{T\theta}$ = Tangential Component of $\vec{V}_{TA}$

$\dot{\vec{\theta}}_{LOS}$ = Aircraft-to-Target Line-of-Sight Slew Rate

The vertical and horizontal plane trajectories for air-to-ground gunnery are shown in Figures 2.4.0.3 and 2.4.0.4, respectively. The vertical and horizontal plane trajectories for air-to-air gunnery are shown in Figures 2.4.0.5 and 2.4.0.6, respectively. For the air-to-ground delivery shown in Figure 2.4.0.3 and 2.4.0.4, the velocity of the target with respect to the ground was assumed to be zero.

2.11

Figure 2.4.0.3--Vertical Plane Trajectory of Air-to-Ground Projectile
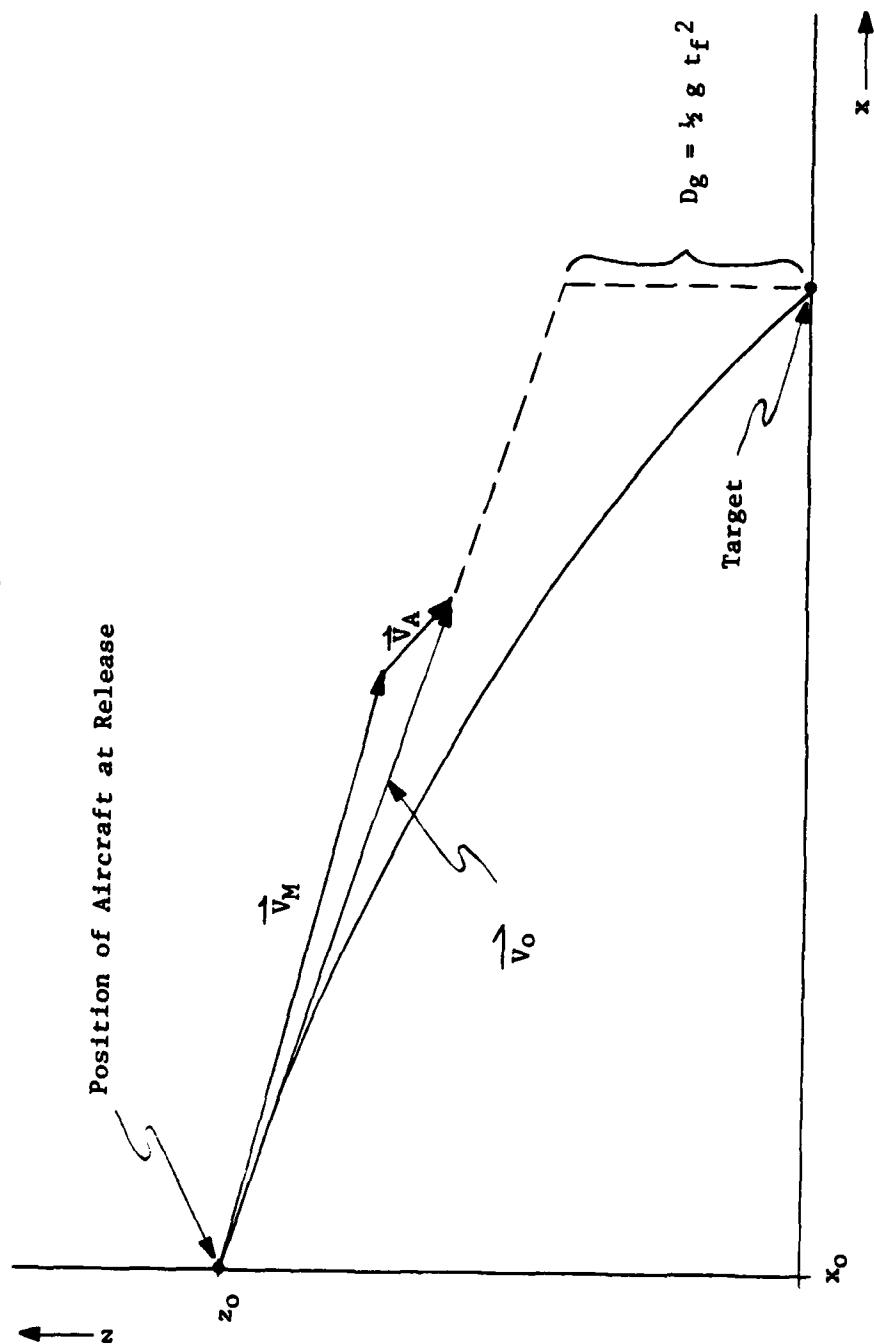
D_g = Gravity Drop Distance
$\overline{V}_o$ = Release Velocity of Projectile
$\overline{V}_A$ = Velocity of Attacking Aircraft
$\overline{V}_M$ = Muzzle Velocity of Projectile

$D_g = \frac{1}{2} g \, t_f^2$

Position of Aircraft at Release

Target

z_o

x_o

x →

z →

2.11a
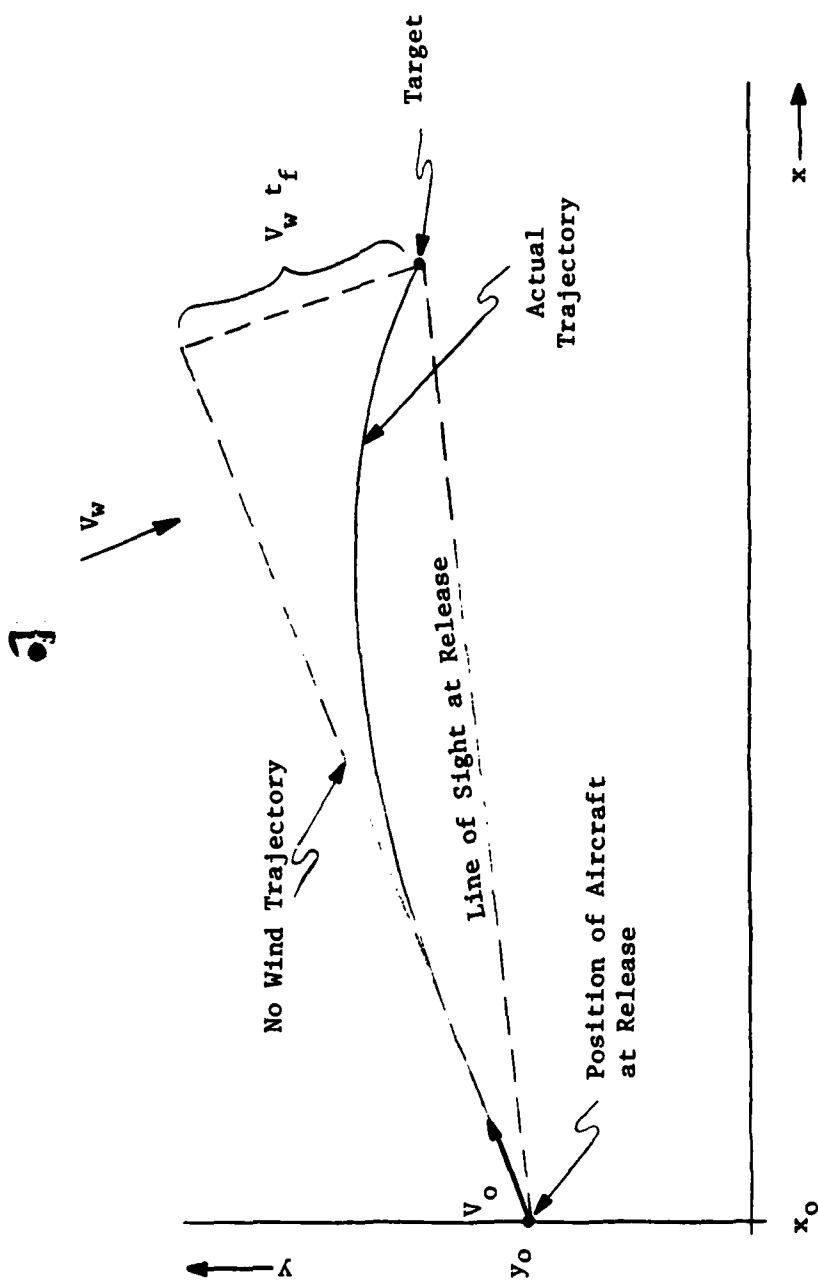
t_f = Time of Flight of Projectile
V_o = Projectile Release Velocity
V_w = Wind Velocity

Figure 2.4.0.4 - - Horizontal Plane Trajectory of
Air-to-Ground Projectile

2.11b

$D_g = \frac{1}{2} g t_f^2$

Position of Target at Impact

Position of Target at Release

V_T

Actual Trajectory

Trajectory without Gravity

R_T

Line-of-Sight

Position of Aircraft at Release

V_o

x_o

z_o

z

x
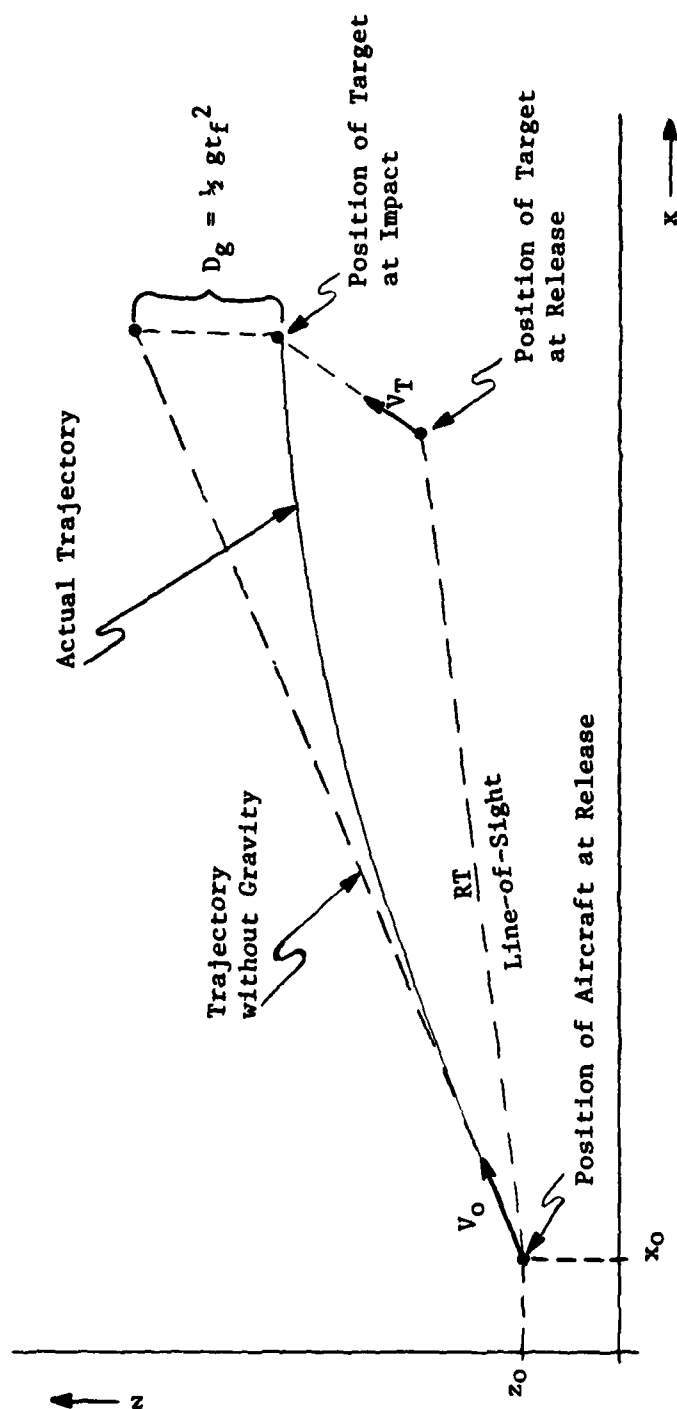
D_g = Gravity Drop Distance

g = Acceleration due to Gravity

R_T = Range to Target

t_f = Projectile Time of Flight

V_o = Projectile Release Velocity

V_T = Target Velocity

Figure 2.4.0.5—Vertical Plane Trajectory of Air-to-Air Projectile

2.11c

R_T = Range to Target
t_f = Projectile Time of Flight
V_o = Projectile Release Velocity
V_T = Target Velocity
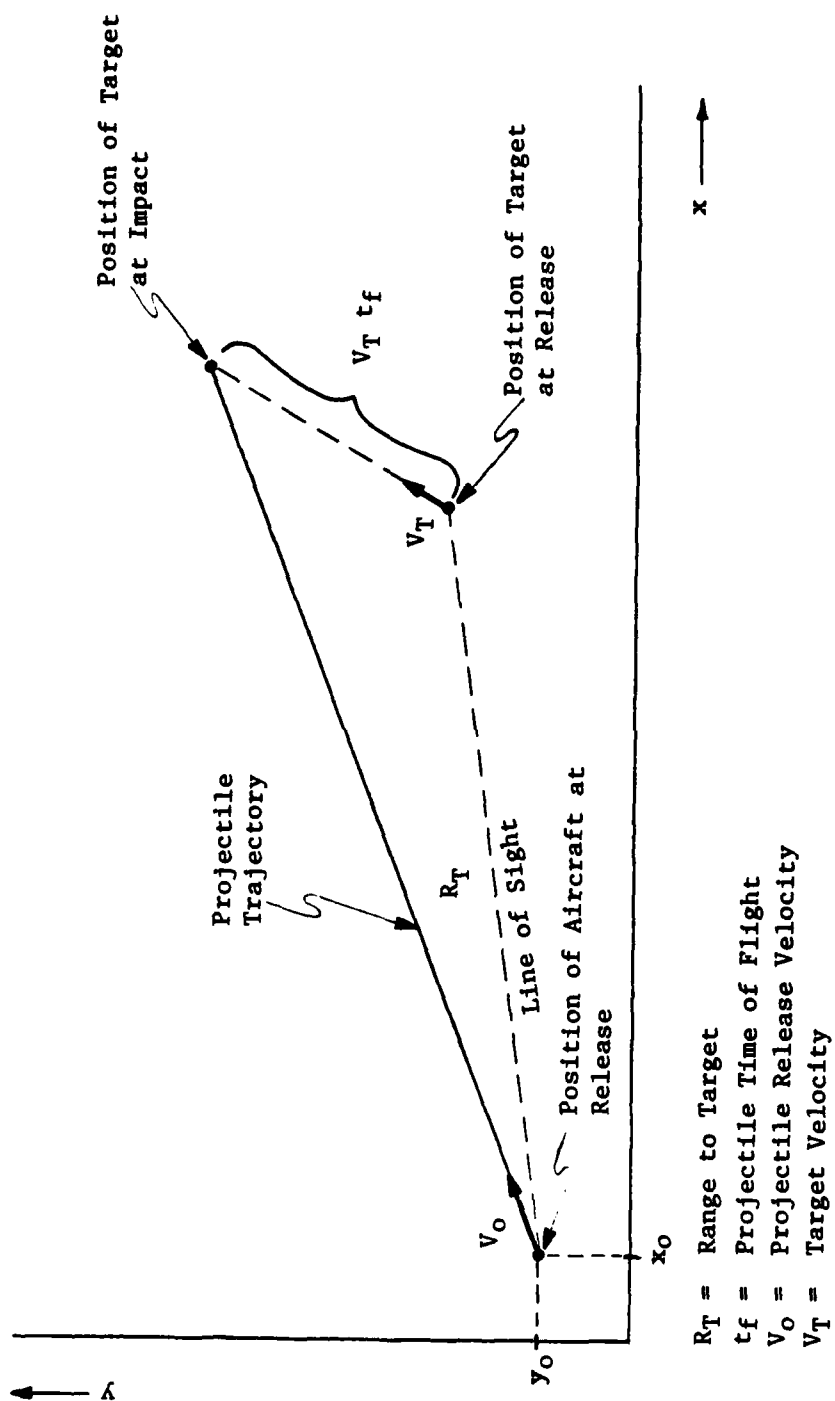
Figure 2.4.0.6—Horizontal Plane Trajectory of Air-to-Air Projectile

2.5 Unguided Rocket Delivery

The factors affecting the trajectory of a rocket differ from those of a bomb or gun projectile in two major respects.

       (1) There exists a time-varying force (thrust) along the longitudinal axis.

       (2) The mass of the rocket changes (decreases) with time.

      (There also are significant normal (non-longitudinal) forces affecting the flight path of a rocket, but their effects, not generally included in the equations of motion, are treated as dispersions on the nominal trajectory.)

The differential equations of motion for an unguided rocket are presented below. They differ from those of a projectile by the inclusion of terms for the components of thrust and by the use of a time-varying mass.

$$\ddot{x} = -\left(\frac{\rho K_D}{2\, m(t)}\right)\, (\dot{x})\, V_T + \left(\frac{F_T(t)}{m(t)}\right)\left(\frac{\dot{x}}{V_T}\right)$$

$$\ddot{z} = -\left(\frac{\rho K_D}{2\, m(t)}\right)\, (\dot{z})\, V_T + \left(\frac{F_T(t)}{m(t)}\right)\left(\frac{\dot{z}}{V_T}\right) - g$$

where:

    $F_T(t)$ = Time-Varying Thrust of Rocket

    $m(t)$ = Time-Varying Mass of Rocket

    V_T = Total Velocity of Rocket with respect to Air Mass

    $V_T \quad = \left[(\dot{x})^2 + (\dot{z})^2\right]^{1/2}$

As for bombs and projectiles, the air density, ρ, is a function of altitude and the drag coefficient, K_D, is a function of mach number.

Figure 2.5.0.1 shows a curve of drag coefficient versus mach number for a 2.75

inch FFAR rocket. Figures 2.5.0.2 and 2.5.0.3 show curves of mass and thrust,

respectively, as a function of time, for the same rocket. (The mass versus

time curve is presented as the ratio of burnout mass to mass because the mass

appears in the denominator of the equations of motion. Similarly, the thrust

information is presented as the ratio of thrust to mass.) Regardless of the

thrust and time varying mass in the equations of motion, the weapon delivery

problem for a rocket is solved in the same manner as that described for bombs

and projectiles. The trajectories for unguided rockets closely resemble those

for projectiles. The same parameters and considerations (wind velocity,

gravity drop, and target motion) apply. For illustrations of unguided rocket

trajectories, the reader is referred to Figures 2.4.0.3 through 2.4.0.6.
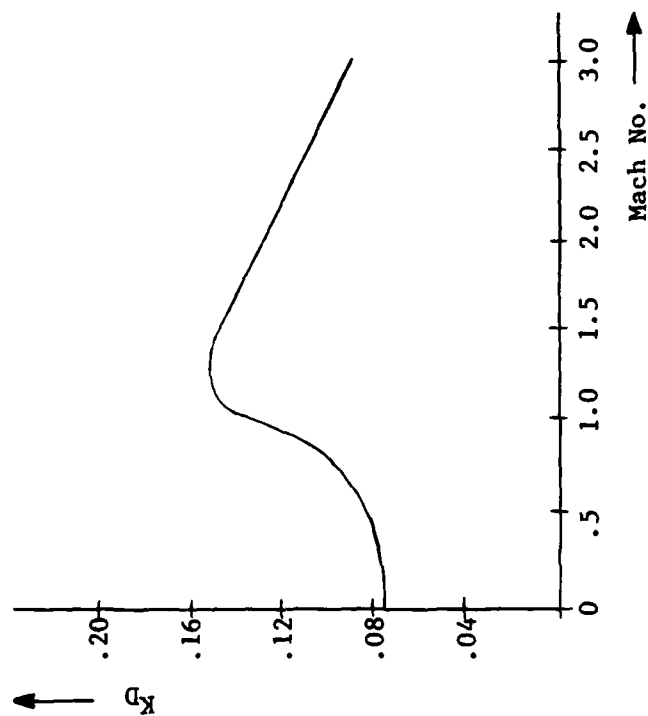
Figure 2.5.0.1—Drag Coefficient Versus Mach Number for a 2.75 Inch FFAR Rocket

2.13a
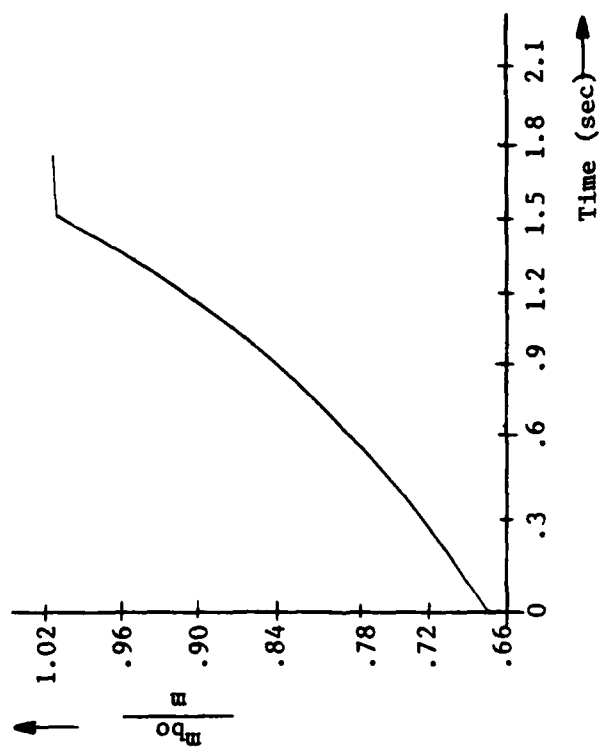
Figure 2.5.0.2--Burnout Mass-to-Mass Ratio for a 2.75 Inch FFAR Rocket

2.13b

$\left(\dfrac{F_T}{m}\right)$ X 10^{-3}

3.0
2.5
2.0
1.5
1.0
0.5
0

0   .3   .6   .9   1.2 1.5 1.8 2.1

Time (sec)
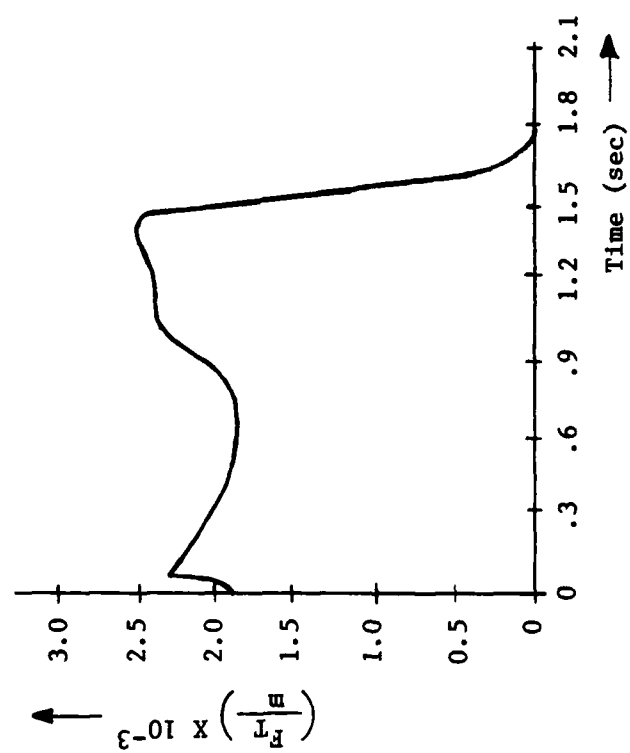
Figure 2.5.0.3--Thrust to Mass Ratio for a 2.75 Inch FFAR Rocket

2.6 Guided Missile Delivery

The principles involved in the delivery of guided missiles are fundamentally different from those involved with unguided missiles. For a guided missile, delivery is accomplished by means of post-release guidance and control rather than precise adjustment of conditions at release. Release conditions must be controlled only to the extent required to place the missile in a position from which it can, with its thrust and control limitations, intercept the target. As a result of post-release steering, the trajectory of a guided missile will depend upon events subsequent to release and will be entirely different from that for an unguided missile, whether thrusting or not. (Note that a guided missile does not necessarily have thrust. In this context, a laser-guided bomb is considered to be a "guided missile".)

2.6.1 Types of Missile Guidance -- The basic types of missile guidance and control are listed and briefly discussed below.

Command Link Guidance -- In command link guidance, the target location and missile aiming error are determined by systems external to the missile. Steering commands are generated and transmitted to the missile by RF, optical, wire, or other type of data link. The principal advantage of command link guidance is the relative simplicity of the missile-borne portion of the system. The principal disadvantages are the relative complexity of the overall system, the fact that accuracy decreases with increasing range from tracker to target, and the fact that the command link, (except for wire guidance), is easily jammed. The Bullpup A (AGM-12B) and the TOW missiles are examples of command-link-guided missiles.

Beam Riding -- A beam riding missile follows a narrow beam of electromagnetic radiation (RF or optical) to the target. The beam is provided by a target-tracking system external to the missile. The principal advantage of a beam-riding missile is the relative simplicity of the overall system. (It is unnecessary for the system to track the missile.) The principal disadvantages are the need to maintain the guiding beam until impact and the fact that accuracy decreases with increasing range from tracker to target. An example of a beam-riding missile is the no-longer-operational Talos.

Semi-Active Homing -- A semi-active homing missile is guided to the target by ("homes on") returns (reflections) from a target illuminated by a source external to the missile. The illumination may be RF, optical, or other radiation and may

be provided by the attacking aircraft, another aircraft, or a ground-based target designating source. The principal advantages of a semi-active homing missile are the fact that accuracy increases as the missile closes on the target and the fact that the missile is passive (does not radiate). The principal disadvantage is the need for continuous illumination of the target by the attacking aircraft or other source. Examples of semi-active homing missiles are the Sparrow (AIM - 7F), the laser-guided bomb (MK 84 LDGP), the Bulldog (AGM - 83A), and the Phoenix (AIM - 54A) (Mid-course guidance phase).

Active Homing -- An active homing missile "homes on" returns from a target illuminated by a source incorporated in the missile. The principal advantage of active homing is its "launch and leave" capability -- that is, the fact that the attacking aircraft need not continue to track, or even illuminate, the target. As for any homing missile, the accuracy of an actively homing missile increases as the missile closes on the target. Furthermore, since the illuminating source is in the missile itself, the strength of the signal also increases as the missile closes on the target. The principal disadvantages of active homing are the fact that the missile-borne equipment is relatively complex (incorporating a full, independent tracking system), and the fact that the missile is not passive and can, therefore, be detected. An example of an active homing missile is the Phoenix (AIM - 54) in the terminal guidance phase.

Passive Homing -- A passive homing missile homes on radiation self-emitted by the target. Such self omissions may be due to natural illumination (light), due to the operation of radiating equipment, or a result of target temperature

2.16

(infrared radiation). The principal advantages of passive homing are the "launch and leave" capability, the non-radiating operation, and the fact that accuracy increases as the missile closes on the target. The principal disadvantage of active homing is its dependence upon target-generated emissions. Examples of passive homing missiles are the Sidewinder (AIM - 9H), the Walleye (AGM - 62A), the Maverick (AGM - 65A), the Shrike (AGM - 54A), and Standard ARM (AGM - 78).

Open-Loop Navigation -- A missile employing open-loop navigation does not sense the current position of the target and correct its course accordingly. It simply flies to a pre-designated location. Open-loop navigation has been employed all the way to missile impact against fixed targets. More frequently, it is used for mid-course guidance to allow target acquisition by a terminal-guidance system. An example of open-loop guidance is the inertial navigation system employed for mid-course guidance in the Standard ARM (AGM - 78).

2.6.2 Guided Missile Steering -- Steering is that process by which guidance system commands are translated into flight control system inputs in order to control the flight path of the missile. There are two principal methods by which missile steering is commonly effected: "pursuit steering" and "proportional navigation".

Pursuit steering generates flight control system inputs intended to produce a flight path directed toward the target at every point in time. That is, pursuit steering attempts to maintain the velocity vector of the missile aligned with the instantaneous missile-to-target line-of-sight. In order to accomplish pursuit steering, the system determines the steering error angle and attempts to generate a missile turn rate proportional to that angle. Thus, the missile makes no attempt to "lead" the target. The result is a trajectory similar to that shown in Figure 2.6.2.1(a). The principal advantage of pursuit steering is its simplicity. Its principal disadvantages are:

(1) Since no attempt is made to "lead" or "head-off" the target, the missile relies entirely upon its speed advantage to overtake the target. Such a trajectory is not only inefficient, but may place the target beyond the thrust or control limitations imposed by missile on-board stores.

(2) As the missile-to-target range decreases, the steering control moment required by pursuit steering increases without limit. The result is an inevitable control inadequacy during the last moments before impact, resulting in a sometimes-unacceptable miss distance.

(3) When the flight path error sensor is fixed to the body of the missile, the angles of attack caused by maneuvering result in a loss of control sensitivity and, possibly, loss of target track.

Proportional Navigation -- The term "proportional navigation" is a misnomer in that the technique is not one of navigation but of steering. With proportional

2.18

(b) Lead Pursuit

Target Path

Initial LOS

T_O

T_I

M_O

V_{M_O}

(a) Pure Pursuit

Target Path

Initial LOS

T_O

T_I

M_O

V_{M_O}
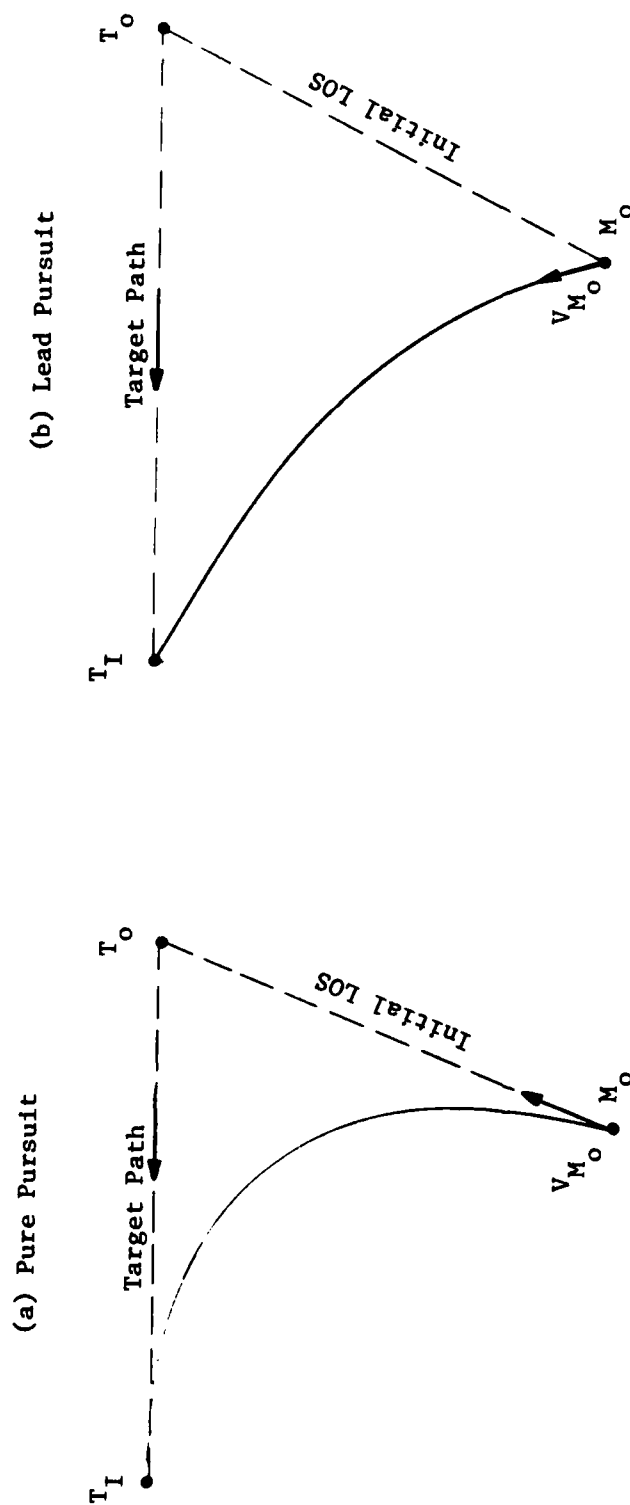
M_O = Missile Position at Release

T_O = Target Position at Release

T_I = Target Position at Impact

V_{M_O} = Missile Velocity at Release

Figure 2.6.2.1 -- Guided Missile Pursuit
Steering Trajectories

2.18a

navigation, the system attempts to generate a flight path that is a collision

course with the target. Thus, the missile "leads" the target. In order to

accomplish such a collision course, the system determines the angular rate of

change of the missile-to-target line of sight and attempts to generate a missile

turning rate proportional to that angular rate. If the constant of propor-

tionality between the line-of-sight slew rate and the missile turning rate

were infinite, the result would be a direct, target-intercepting flight path

for which the angle between the missile velocity vector and the missile-to-

target line-of-sight would remain constant, as shown in Figure 2.6.2.2. In

an actual system, the constant of proportionality between the line-of-sight

slew rate and the missile turning rate is less than infinity, and produces a

missile flight path similar to those illustrated in Figure 2.6.2.3. Even if

an extremely large proportionality constant were possible, it would not be

desirable because the resulting hyperactivity of the missile control system

would greatly increase the induced drag of the missile (thereby reducing the

effective range of the missile) and would also prematurely exhaust the available

missile control energy. Most proportional navigation systems employ a propor-

tionality constant between three and five. A value in that range avoids ex-

cessive control action while providing sufficient lead in the missile flight

path to nearly optimize the missile trajectory. Even with a maneuvering target,

proportional navigation provides major advantages over pursuit steering. For

a target turning at a constant rate, the lateral acceleration of a missile

with proportional navigation is actually less than that of the target (if the

missile has a speed advantage). For the above reasons, the vast majority of

modern airborne guided missiles employ proportional navigation steering.
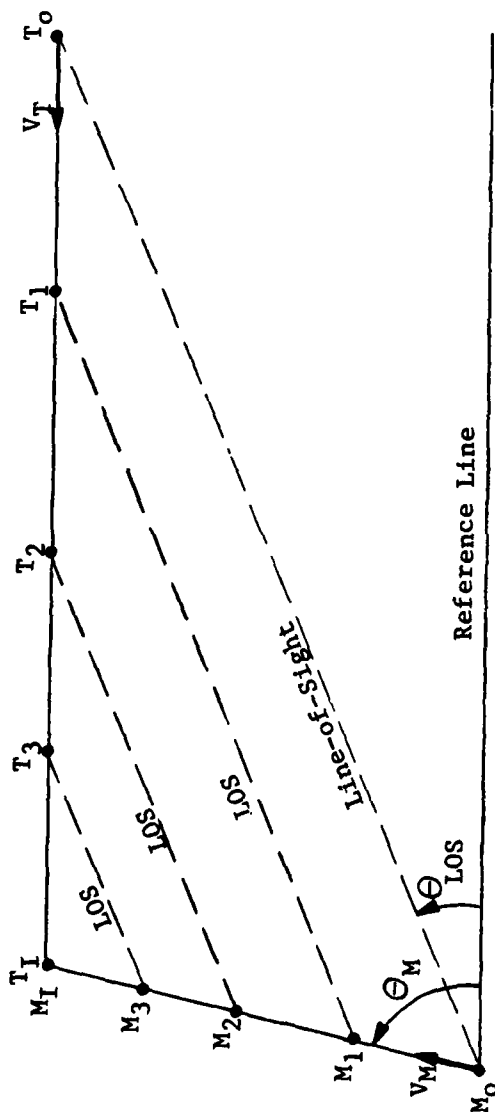
Proportional Navigation Control Law:

$\Theta_M = K\Theta_{LOS}$

For the non-maneuvering target shown:

$\Theta_M = \Theta_{LOS} = o$

M_i = Missile Position at Time t_i

T_i = Target Positon at Time t_i

V_M = Velocity Vector of Missile

V_T = Velocity Vector of Target

Θ_M = Flight Path Angle of Missile

Θ_{LOS} = Missile-to-Target Line of Sight Angle

Reference Line

Line-of-Sight

LOS

T_o

T_1

T_2

T_3

T_I

M_I

M_3

M_2

M_1

M_o

V_T

V_M

Θ_M

Θ LOS

Figure 2.6.2.2--Proportional Navigation Control Law

2.19a

Initial Position
of Target

V_T T_o

$\dot{\theta}_M = K \dot{\theta}_{LOS}$
$\left(V_M = 2 \ V_T \right)$

Impact Points

Initial LOS

1
2
3
6

$K = \infty$

Initial Position
of Missile

M_o

V_M = Velocity of Missile
V_T = Velocity of Target
K = Constant of Proportionality
θ_M = Missile Flight Path Angle
θ_{LOS} = Line-of-Sight Angle
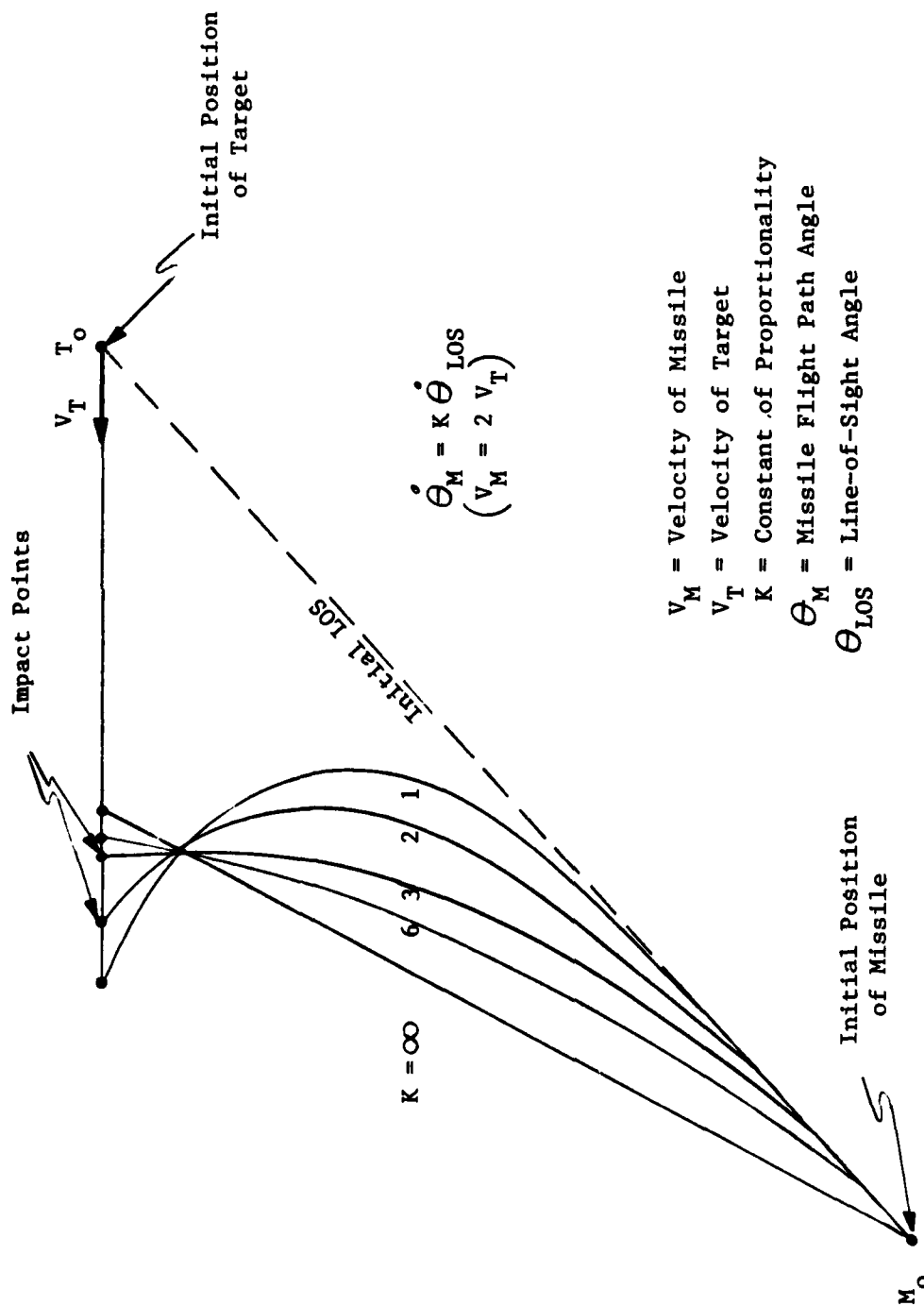
Figure 2.6.2.3 - - Proportional Navigation Trajectories for a Non-Maneuvering Target

2.19b

3.0 Airborne Weapon System Performance Characteristics

3.1 The Weapon Delivery Error Model

3.1.1 Derivation of the Error Model -- The weapon system characteristics that
contribute to weapon delivery error include not only those characteristics
associated with the integrated weapon delivery computations and functions per
se, but also those characteristics associated with the individual weapon system
sub-systems. In order to determine all of the relevant weapon system character-
istics, the test and evaluation process is performed at two distinct levels.
At the first level, the overall (integrated) weapon system performance character-
istics are determined. For unguided weapon delivery, this level entails the
determination of the errors in the weapon release conditions that produced the
observed target miss distances. At the second level, the characteristics of the
individual weapon system sub-systems are determined. For unguided weapon
delivery, this level entails the determination of the sub-system errors that pro-
duced the observed errors in weapon release conditions. The material in this
section is concerned with the error model employed at the first level--that
associated with integrated weapon system characteristics. The error models
associated with the characteristics of the individual sub-systems are discussed
in the texts concerned with those sub-systems.

The integrated weapon system error model is derived directly from the missile
differential equations of motion and wind-correction equations presented in
Section 2.2 of this text. More precisely, the error model for a specific weapon
is derived from the solutions to (integrals of) the particular sub-set of those

3.1

equations applicable to that specific weapon. In general, the solutions to

the applicable differential equations of motion are of the form:

$$X_f = \int_{t_r}^{t_f}\int \ddot{x}(t)\, dt\, dt \;+\; V_{xr}\left[t_f - t_r\right] \;+\; x_r$$

$$Y_f = \int_{t_r}^{t_f}\int \ddot{y}(t)\, dt\, dt \;+\; V_{yr}\left[t_f - t_r\right] \;+\; Y_r$$

where:

$X(t)$ = Down-Range Missile Position

$Y(t)$ = Cross-Range Missile Position

V_x = Down-Range Missile Velocity

V_y = Cross-Range Missile Velocity

t_r = Time of Release

t_f = Time of "Impact"

X_f = $X(t_f)$

Y_f = $Y(t_f)$

X_r = $X(t_r)$

Y_r = $Y(t_r)$

V_{xr} = $V_x(t_r)$

V_{yr} = $V_y(t_r)$

and where $\ddot{x}(t)$, $\ddot{y}(t)$, and t_f are functions of the initial conditions in three

directions, the prevailing wind, and various sytem parameters. From these

equations, error model equations can be derived, of the form:

3.2

$$\Delta X_f = \left(\frac{\partial X_f}{\partial X_r}\right) \Delta X_r + \left(\frac{\partial X_f}{\partial V_{xr}}\right) \Delta V_{xr} + \left(\frac{\partial X_f}{\partial C_1}\right) \Delta C_1$$

$$+ \left(\frac{\partial X_f}{\partial C_2}\right) \Delta C_2 + \left(\frac{\partial X_f}{\partial V_{xw}}\right) \Delta V_{xw}$$

where ΔX_f is error in X_f (miss distance), ΔX_r and ΔV_{xr} are errors in the required release conditions, ΔC_1 and ΔC_2 are errors in system parameters, and ΔV_{xw} is error in the determination of down-range wind. In some cases, the sensitivities (partial derivatives) are calculated from analytic relationships. More frequently, they are evaluated numerically by computationally integrating the differential equations of motion. The integrated system error model allows the system evaluator to identify the effects of various errors in the weapon delivery computations, release conditions, and wind velocity on miss distance. It does not assist him in identifying the internal sub-system contributors to the errors in release conditions. For that task, individual sub-system error models are required.

3.1.2 Sources of Error -- The major contributors to weapon delivery error are errors in the determination of the quantities listed below and discussed in the following paragraphs.

> Aircraft-to-Target Range
> Target Line-of-Sight Angles
> Aircraft-to-Target Velocity
> Target Motion
> Wind Velocity
> Computed Weapon Trajectory
> Aircraft Altitude
> Aircraft True Airspeed
> Aircraft Angle of Attack
> Aircraft Angle of Sideslip
> Aircraft Ground Speed
> Aircraft Heading
> Aircraft Attitude
> Weapon System Alignment
> Pilot Aiming and Steering
> Weapon Separation Delay
> Weapon Separation Velocity
> Weapon Characteristic Anomalies

Aircraft-to-Target Range -- Aircraft-to-target range is determined either by direct measurement of slant range by means of a radar or laser ranger; or, (for air-to-ground delivery only), indirectly from measurements of aircraft-to-target altitude difference and line-of-sight depression angle. The contributing error sources are, therefore, either the ranging device or the altitude and target LOS angle-measuring devices.

Target Line-of-Sight Angles -- As implied above, target line-of-sight angle errors can contribute to both target bearing errors and to target range errors, depending on the ranging method employed by the system.

Aircraft-to-Target Velocity -- The velocity of the target with respect to the aircraft is determined in order to compute target velocity with respect to the

air mass (by adding the velocity of the aircraft with respect to the air mass). The target velocity is required in order to predict its position at point of impact; that is, in order to compute the necessary "lead". Aircraft-to-target velocity is generally determined by the ranging sensor or by Doppler measurements.

Target Motion -- Unanticipated target motion is a major source of weapon delivery error. In an effort to minimize this error, some tracking systems measure not only the target position and velocity, but also its acceleration.

Wind Velocity -- Wind velocity is usually determined indirectly as the vector difference between aircraft ground velocity and true airspeed. Ground velocity is determined by the navigation system; true airspeed is determined by the air data system. Wind velocity is employed in the weapon delivery transformation from air mass to ground coordinates for air-to-ground delivery.

Computed Weapon Trajectory -- Differences between the computed and actual weapon trajectories fall into two categories: those caused by errors in the computed trajectory and those caused by anomalous weapon characteristics. Errors of the first type can be identified by comparing the impact point predicted by the weapon delivery computer with that predicted by a reference (nominal) weapon trajectory simulation using the same initial (release) conditions. Errors of the second type can be identified by comparing the actual (observed) impact point with that predicted by the reference trajectory simulation with initial conditions based upon measurements of the actual release conditions.

Aircraft Altitude -- Aircraft altitude at weapon release is employed by the weapon delivery system as an initial condition in predicting the weapon trajectory (and air density) and, in some systems, in determining the slant range to a ground target.

Aircraft True Airspeed -- Aircraft true airspeed is employed by the weapon delivery system as an initial condition in predicting the weapon trajectory (and drag coefficient) and, in air-to-ground weapon delivery, in determining wind velocity.

Aircraft Angle of Attack -- Aircraft angle of attack is employed by a weapon delivery system in determining the direction of the true airspeed vector (flight path angle).

Aircraft Angle of Sideslip -- Aircraft angle of sideslip at weapon release is usually assumed to be zero for weapon delivery. It must be determined in order to verify the zero value at release.

Aircraft Ground Speed -- The velocity of the aircraft with respect to the ground is employed by a weapon delivery system in determining the wind vector.

Aircraft Heading -- Aircraft heading is employed by some weapon delivery systems in determining the direction of the ground velocity vector.

Aircraft Attitude -- The pitch angle of the aircraft is employed by a weapon delivery system in determining the direction of the true airspeed vector (flight path angle).

Weapon System Alignment -- Errors in the mutual alignment of all weapon delivery sub-systems contribute to directional errors at weapon release.

Pilot Aiming and Steering -- Pilot aiming and steering errors produce directional errors similar to those produced by any other weapon delivery sub-system.

Weapon Separation Delay -- All weapon delivery systems exhibit some delay between the weapon release command and actual weapon separation. A nominal value for this delay is included in the weapon trajectory computations. When the actual separation delay time deviates from the assumed (nominal) value, an error in weapon release conditions occurs.

Weapon Separation Velocity -- Weapons, including bombs, are separated from the aircraft with a nominal separation velocity with respect to the aircraft. When the actual separation velocity deviates from the nominal value, an error in weapon release velocity occurs.

3.2 Other Weapon Delivery System Characteristics

In addition to the quantitative performance characteristics represented by
the error model, a weapon delivery system exhibits other important functional
characteristics. A list of the most important of these characteristics is
presented below.

    Target Range Limitations
    Target Bearing Limitations
    Target Velocity Limitations
    Aircraft Maneuvering Limitations
    Aircraft Altitude Limitations
    Aircraft Velocity Limitations
    Weather Limitations
    Illumination Limitations
    Weapons Available
    Sensors Available
    Weapon Delivery Modes
    Controls and Displays Effectiveness
    Interfaces with Other Systems
    Electromagnetic Compatibility
    Carrier Suitability
    Weapon Separation Characteristics

3.3 Typical Weapon Delivery System Characteristics

For a description of typical, currently-operational weapons systems, the reader is referred to the separate, classified volume devoted to airborne system hardware descriptions.

4.0 Weapon Delivery System Test and Evaluation

This section is concerned with test methods peculiar to integrated weapon delivery system parameter determination and performance testing. Topics concerned with the testing of the individual sub-systems of a weapons system are discussed in the texts devoted to those individual sub-systems.

In the following paragraphs, a brief description is given of the methods employed to determine system compliance with the overall weapon delivery system accuracy specifications. General testing, such as environmental, electromagnetic compatibility, reliability, and maintainability testing, is discussed in a separate text concerned with tests common to all airborne systems.

4.1 Weapon Delivery Accuracy Testing

4.1.1 The Basic Method of Testing -- The primary objective of weapon delivery system accuracy testing is the determination of the overall accuracy with which an integrated weapon system delivers weapons. A secondary, but closely related, objective is the determination of an "error budget"; that is, the identification of the source of error. Without such identification, a meaningful interpretation of the error is impossible. For example, two systematic errors, of equal magnitude and opposite sign under the conditions of the test, can cancel, thus concealing their presence. Under other conditions, the errors may not cancel, thereby providing large weapon delivery errors. The only assurance that such errors do not exist is the construction of an error budget.

In order to obtain sufficient information to identify the major error contributors, listed in Section 3.1.2 of this text, test measurements must be made of not only the weapon miss distances, but also the various inputs to the weapon delivery computations, the results of the weapon delivery computations (the computed weapon release conditions), and the actual weapon release conditions. Measurements of these quantities, when used in conjunction with an error model as discussed in Section 3.1.1 of this text, allow the construction of an error budget.

4.1.2 No-Drop Weapon Delivery Scoring -- In evaluating the accuracy of the

weapon delivery system itself, the effects of weapon ballistics are not of

direct interest. In fact, they make the task of weapon delivery system evalua-

tion more difficult. (They must be removed from the data before weapon delivery

system evaluation can be performed). For that reason, there is inherent ad-

vantage in a method of testing that avoids the effects of anomalous weapon

trajectories on the data. Such a system is no-drop weapon scoring. With

adequate instrumentation, the release conditions can be determined with

sufficient accuracy to allow almost complete evaluation of weapon delivery

system performance, without actually releasing the weapon. (A no-drop weapon

test does not, of course, evaluate the weapon separation process, including

the effects of weapon separation velocity anomalies and anomalies in the small,

but significant, time delay between the re ease command and actual weapon

separation.) By eliminating the often sizeable errors associated with missile

ballistics, no-drop weapon scoring can facilitate the task of determining the

error-contributing characteristics of the weapon delivery system itself.

4.1.3 Weapon Delivery Flight Test Procedure

   (a) Scenario -- Fly prescribed flight path toward instrumented target.
       Designate target to tracking system. Perform prescribed weapon
       delivery maneuver while tracking target. Enable weapon release.
       Continue tracking target through weapon release.

   (b) Test Data -- Track test aircraft, through weapon release, employing
       "range" instrumentation (radar, theodolites, DME, etc.). Determine
       aircraft release conditions (position, velocity, attitude, etc.)
       employing both external and on-board instrumentation. Determine
       and record the weapon delivery parameters, listed in Section 3.1.2
       of this text, at time of release. (Designate time of weapon-release
       by recorded tone or other discrete.) Track weapon trajectory and
       impact employing on-board and/or external instrumentation.

   (c) Data Reduction -- Compare actual weapon trajectory and/or impact
       position, (truth data determined by range instrumentation), with
       position of target, to determine miss distance. Compare actual
       weapon trajectory and/or impact position with impact position
       predicted by reference trajectory simulation to determine weapon
       characteristic anomalies. Compare weapon trajectory and/or impact
       position computed by weapon delivery system with trajectory and/or
       impact position predicted by reference trajectory simulation,
       (truth data), to determine errors in weapon delivery system trajectory
       computations. Compare weapon release conditions indicated by weapon
       delivery system sub-systems with actual weapon release conditions,
       (truth data determined by test instrumentation), to obtain errors in
       sub-system inputs to weapon delivery computations.

4.2 Weapon Delivery Error Scoring

4.2.1 The Ground and Scoring Planes -- Weapon delivery errors (miss distances) are measured in two planes: the ground plane and the scoring plane. The ground plane, shown in Figure 4.2.1.1, is a plane parallel to the surface of the earth (horizontal) and passing through the target. The scoring plane, also shown in Figure 4.2.1.1, is a plane perpendicular to the aircraft-to-target line of sight at release, and passing through the target. Weapon delivery errors in the ground plane are expressed as linear measure (feet or meters). Weapon delivery errors in the scoring plane are generally expressed as angular measure (milliradians).

Point at which Weapon Pierces
Scoring Plane

Weapon Impact
in Ground Plane

Scoring Plane

Ground Plane

Target

LOS

Weapon Release
Point
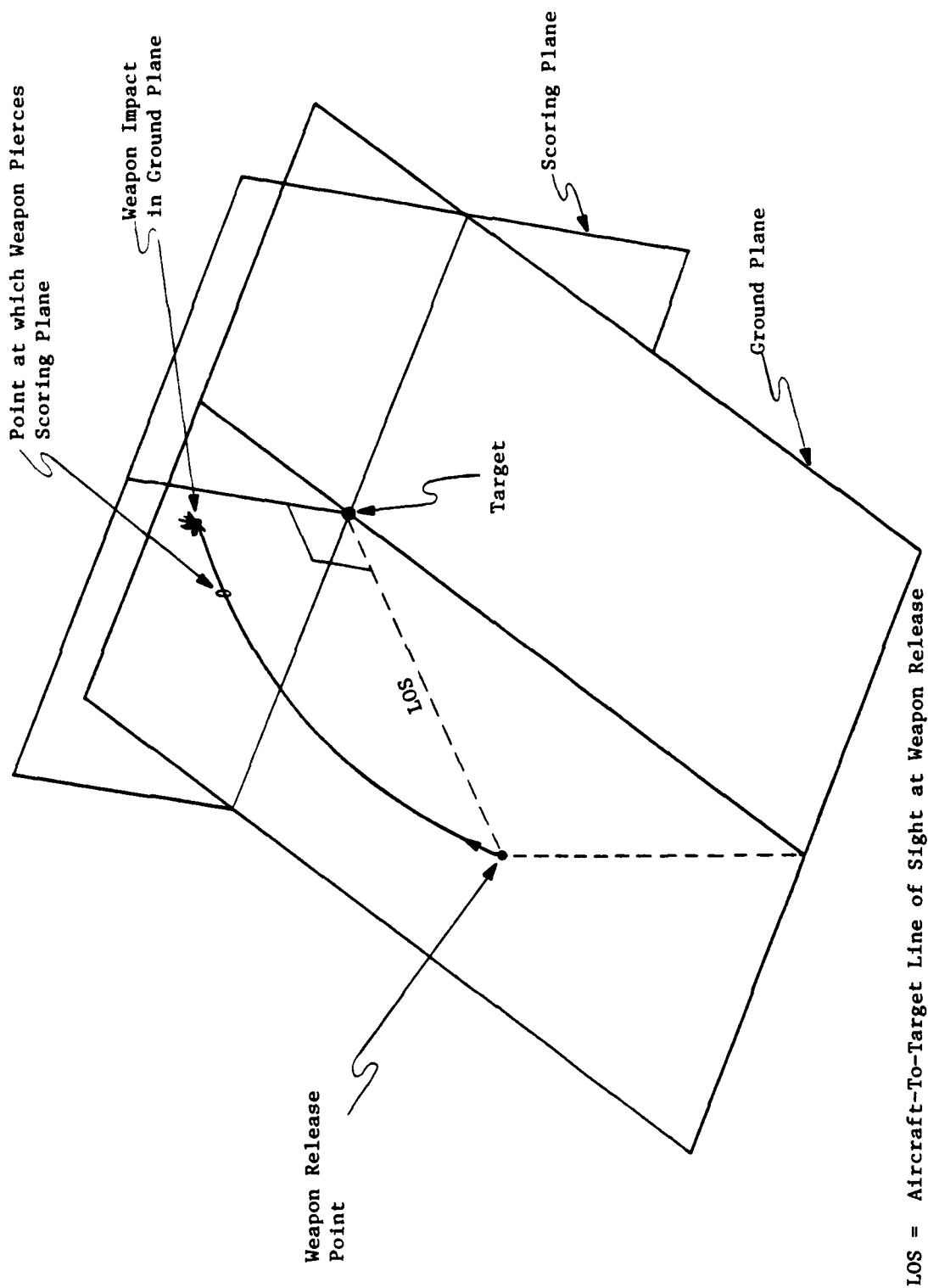
Figure 4.2.1.1 -- Weapon Ground and Scoring Planes

LOS = Aircraft-To-Target Line of Sight at Weapon Release

4.2.2 Air-to-Ground Weapon Delivery Error Scoring -- Air-to-ground weapon

delivery errors (miss distances) are generally measured in the ground plane

as shown in Figure 4.2.1.2. The linear miss distances in range and deflection

(cross-range) are designated therein as E_r and E_d, respectively. Air-to-ground

weapon delivery errors also can be measured in the scoring plane as shown in

Figure 4.2.1.3. The angular miss distances are computed by means of the

relationships:

$$\delta_d = Tan^{-1} \left[Dd / R_T \right] \ (Mils)$$

$$\delta_r = Tan^{-1} \left[Dr / R_T \right] \ (Mils)$$

where:

D_d = Projection of Deflection Miss Distance into Scoring Plane (Meters)

D_r = Projection of Range Miss Distance into Scoring Plane (Meters)

R_T = Range from Weapon Release Point to Target (Meters)

δ_d = Angular Weapon Delivery Error in Deflection (Milliradians)

δ_r = Angular Weapon Delivery Error in Range (Milliradians)

As indicated in Figure 4.2.1.3, D_d and D_r are derived by determining the point at

which a line drawn between the weapon release point and the weapon impact point

pierces the scoring plane. Note that this method of weapon scoring does not

indicate the distance between the weapon and the target at the point of closest

approach.

Point at which LOS to Impact Point
Pierces Scoring Plane

Weapon Impact in
Ground Plane

Target

D_d

D_r

δ_d

R_T

δ_r

Weapon Release
Point

δ_d = Angular Weapon Delivery Error in
Deflection (Mils)

δ_r = Angular Weapon Delivery Error in
Range (Mils)

D_d = Projection of Deflection Miss Distance into Scoring Plane (Meters)

D_r = Projection of Range Miss Distance into Scoring Plane (Meters)

R_T = Range from Release Point to Target (Meters)

Figure 4.2.1.3 -- Air-To-Ground Weapon Scoring in Scoring Plane

4.6b

4.2.3 Air-to-Air Weapon Delivery Error Scoring -- Air-to-air weapon delivery errors are measured in the scoring plane as shown in Figure 4.2.1.4. The angular weapon delivery errors in azimuth and elevation, δ_a and δ_e, respectively, are computed by means of the relationships:

$$\delta_a = Tan^{-1}\left[D_a/R_T\right] \quad (Mils)$$

$$\delta_e = Tan^{-1}\left[D_e/R_T\right] \quad (Mils)$$

where:
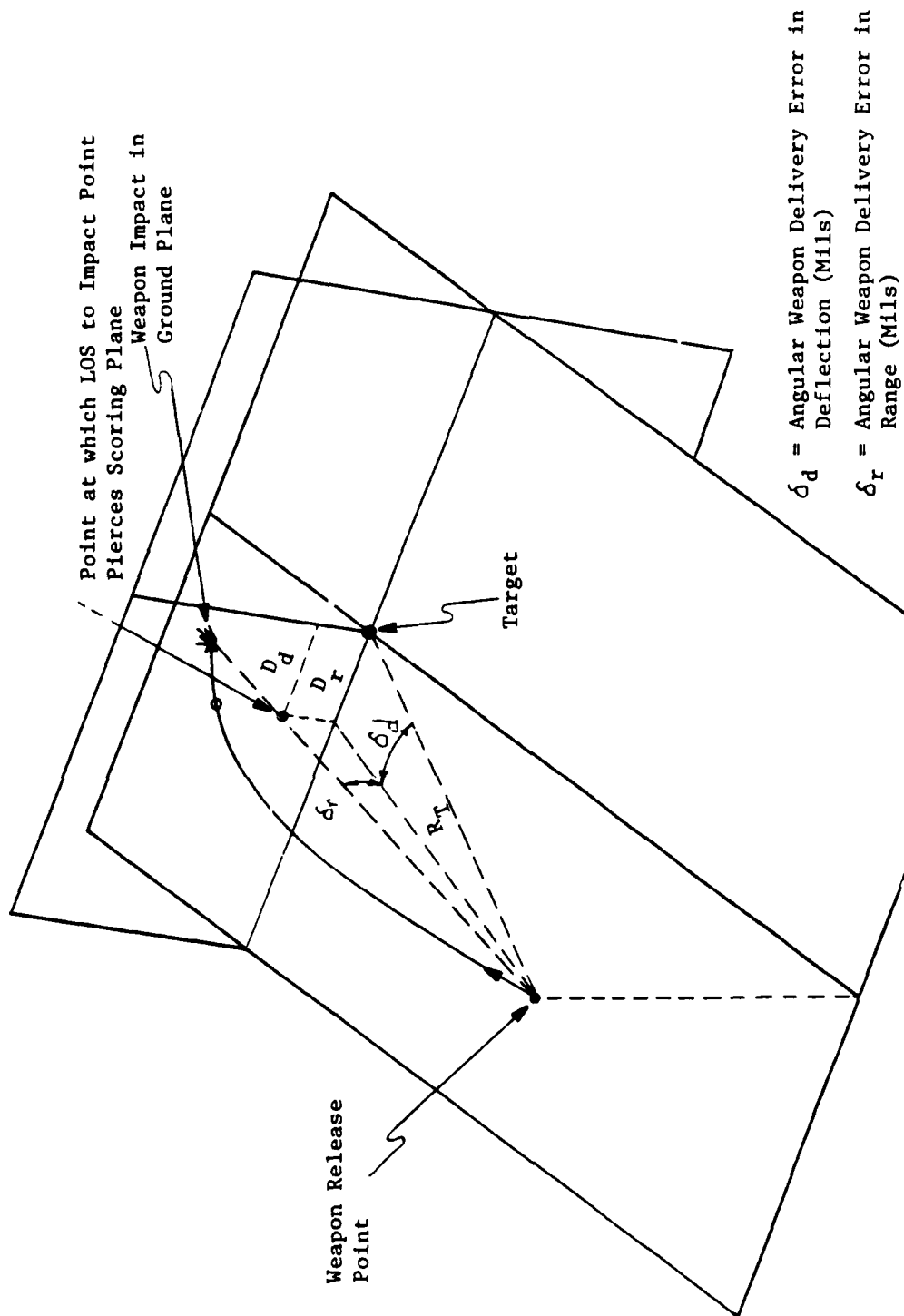
D_a = Azimuth Miss Distance in Scoring Plane (Meters)

D_e = Elevation Miss Distance in Scoring Plane (Meters)

R_T = Range from Weapon Release Point to Target (Meters)

δ_a = Angular Weapon Delivery Error in Azimuth (Milliradians)

δ_e = Angular Weapon Delivery Error in Elevation (Milliradians)

Point at which Weapon Pierces Scoring Plane

Target

D_a

D_e

δ_e

δ_a

R_T

Weapon Trajectory

Weapon Release Point

D_a = Azimuth Miss Distance in Scoring Plane (Meters) δ_a = Angular Weapon Delivery Error in Azimuth (Mils)

D_e = Elevation Miss Distance in Scoring Plane (Meters) δ_e = Angular Weapon Delivery Error in Elevation (Mils)

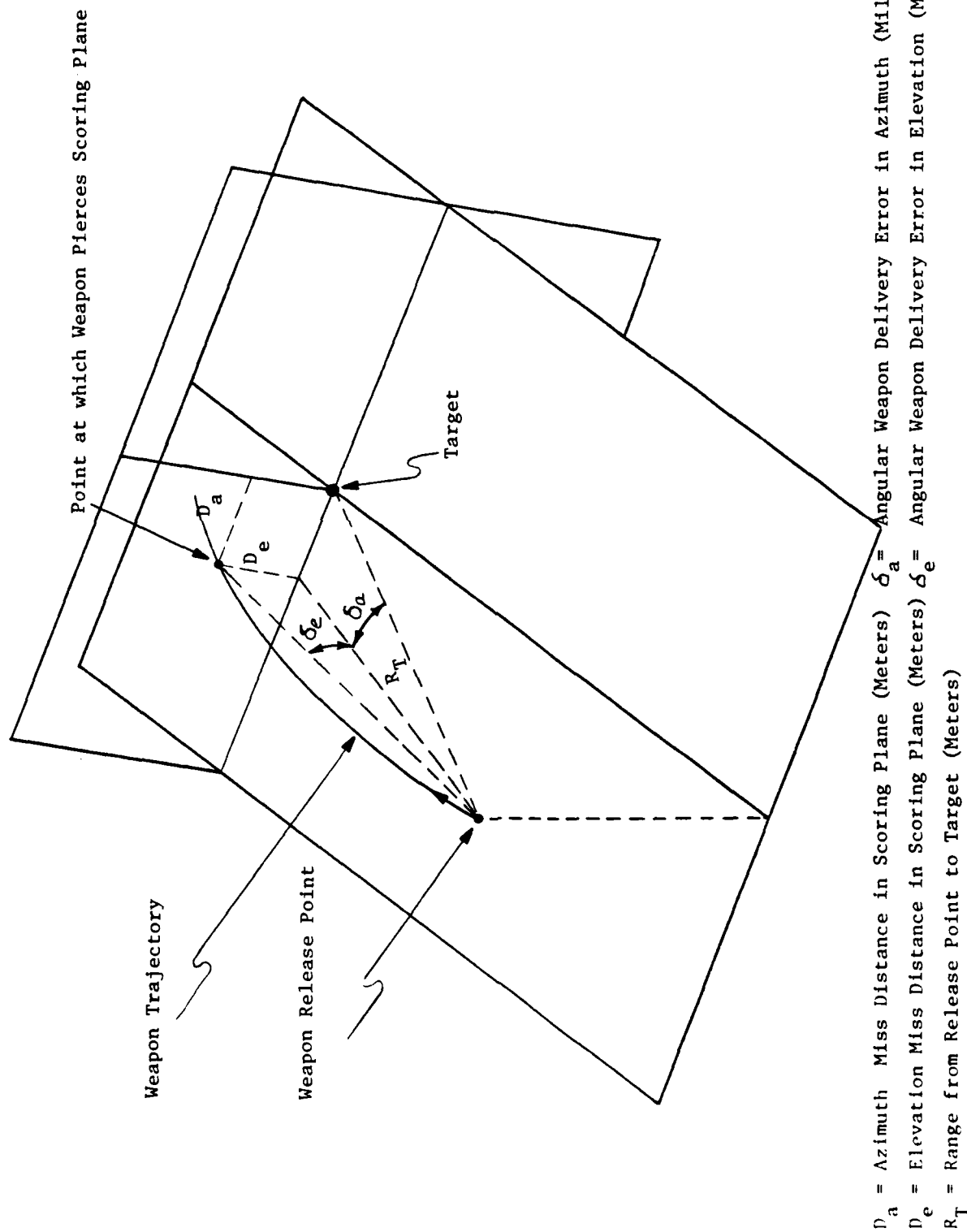R_T = Range from Release Point to Target (Meters)

Figure 4.2.1.4 -- Air-To-Air Weapon Scoring in Scoring Plane

4.7A

4.3 Weapon Delivery Error Data Reduction

Weapon delivery accuracy testing is a stochastic process -- that is, random

errors exist in both the measurements and the quantities being measured. For

that reason, the test results must be presented in terms of the statistics of

the weapon delivery errors measured. In order to obtain the necessary data for

statistical analysis, repeated measurements must be made in each weapon delivery

mode. The results of the tests are then combined to obtain a measure of the

average (mean) weapon delivery error and a measure of the dispersion (variation)

of the weapon delivery error.

The measure of the mean generally employed in weapon delivery accuracy testing

is the Mean point of Impact (MPI). The MPI is defined as that point in the

ground or scoring plane located at the average values of the range (on-track)

and deflection (cross-track) errors, $\overline{E}_r$ and $\overline{E}_d$, respectively. $\overline{E}_r$ and $\overline{E}_d$ are thus

defined by the equations:

$$\overline{E}_r = \frac{1}{N} \sum_{n=1}^{N} (E_r)_i$$

$$\overline{E}_d = \frac{1}{N} \sum_{n=1}^{N} (E_d)_i$$

where $(E_r)_i$ and $(E_d)_i$ are individual weapon delivery errors and N is the total

number of error measurements. (It is common practice to treat multiple-missile

bursts as single weapon deliveries. In such cases, $(E_r)_i$ and $(E_d)_i$ represent

the range and deflection errors of the centroid of the burst impact pattern).

4.8

The measures of weapon delivery error dispersion generally employed are the Range Error Probable (REP), Deflection Error Probable (DEP), and the Circular Error Probable (CEP). The REP is defined as the range interval, centered on either the target or the MPI, that encompasses one-half of the individual impact points. The DEP is defined as the deflection (cross-range) interval, centered on either the target or the MPI, that encompasses one-half of the individual impacts points. The CEP is defined as that circle in the ground or scoring plane, centered on either the target or the MPI, that encloses one-half of the individual impact points. For impact patterns for which the REP and DEP are significantly different, an Elliptical Error Probable (EEP) is sometimes a more useful measure of weapon delivery error dispersion than is the CEP. Typical weapon impact patterns, with the MPI, REP, DEP, and CEP indicated, are shown in Figures 4.3.0.1 and 4.3.0.2, for the ground plane and the scoring plane, respectively.

When the statistical distribution of the weapon delivery errors can be described by Normal (Gaussian) statistics, the REP, DEP, CEP, and EEP can be related to the sample standard deviation of the errors by means of the following equations.

$$REP = 0.6745 \; S \; (E_r)$$

$$DEP = 0.6745 \; S \; (E_d)$$

$$CEP = 1.177 \left[\frac{S(E_r) + S(E_d)}{2} \right]$$

The CEP defined by the above equation represents an approximation to the EEP when the REP and DEP are not equal.
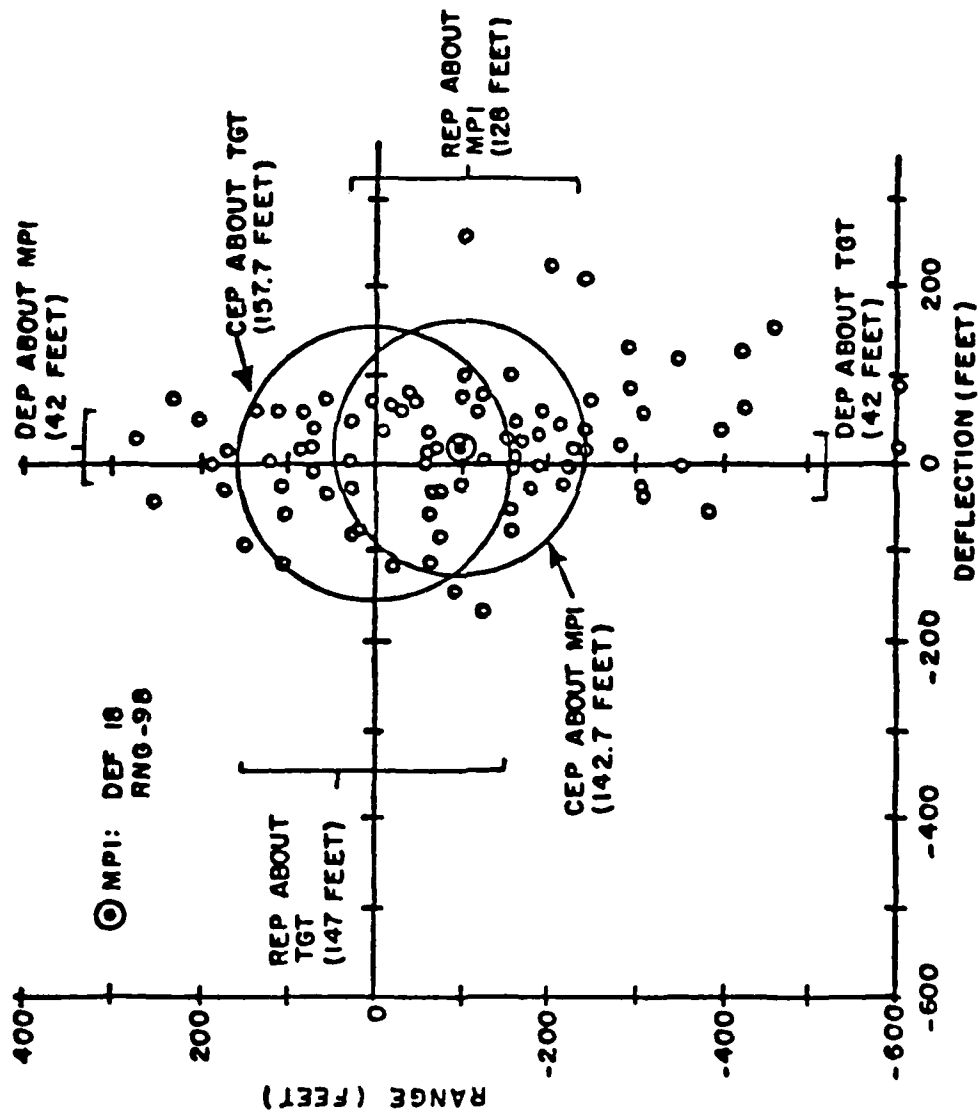
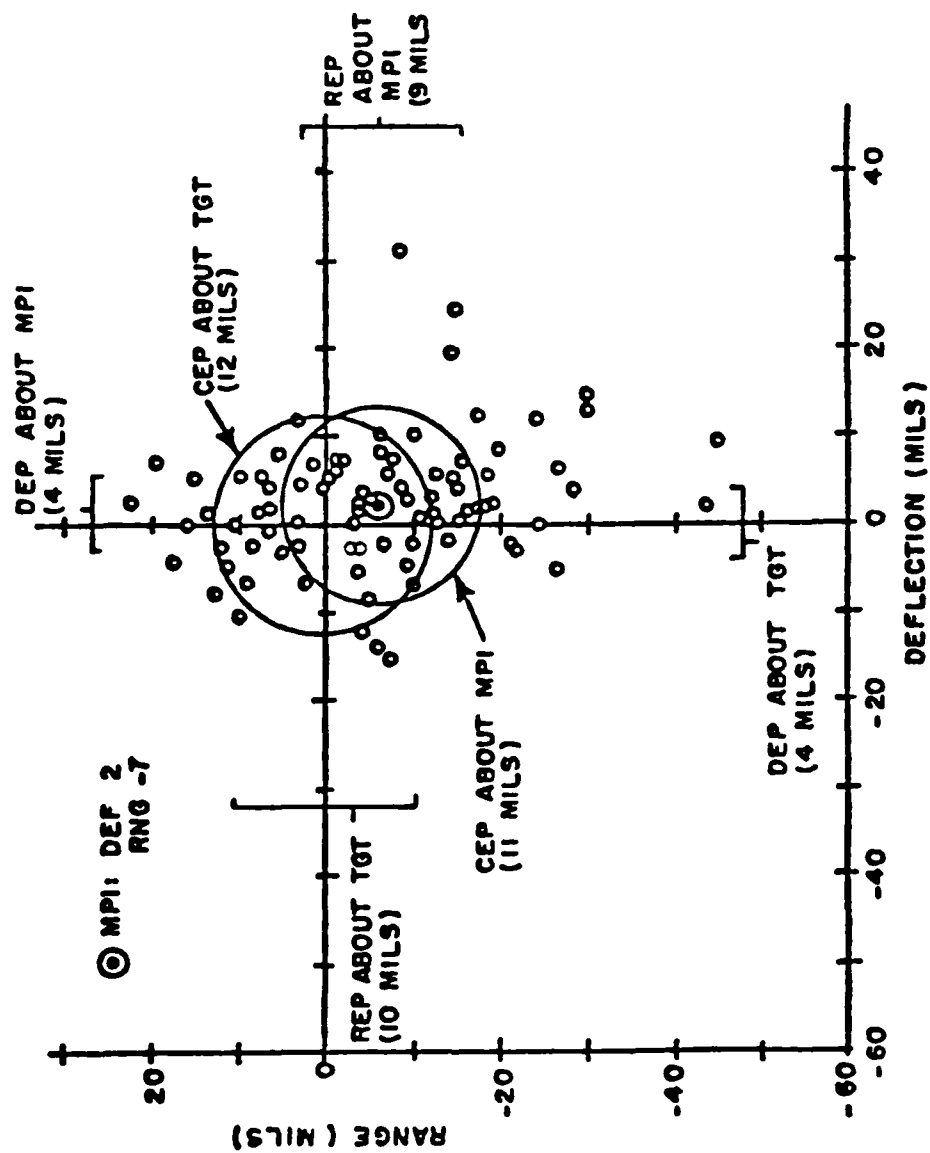Figure 4.3.0.1 -- Typical Ground Plane Weapon Delivery Pattern

4.9a

Figure 4.3.0.2 -- Typical Scoring Plane Weapon Delivery Pattern

4.9b

5.0 Airborne Systems Performance Test and Evaluation

5.1 The Philosophy of Testing

5.1.1 Stages of Testing -- Testing can be categorized as developmental,
functional, or operational, depending upon the stage of development of the test
item. Developmental testing is concerned with the evaluation of design features
for the purpose of design development. The end result of developmental testing
is the proposed final design. Functional testing is concerned with the perfor-
mance evaluation of the final design as a whole. The principal method of evalu-
ation is the quantitative measurement of the ability of the test item to perform
its intended functions. The end result of functional testing is final design
acceptance or rejection. Operational testing is concerned with the evaluation
of the final design and production implementation of the test item. Of primary
interest is the ability of the test item to accomplish its intended operational
mission. The end result of operational testing is acceptance or rejection of the
test item for service use and the recommendation of operational procedures.

5.1.2 Testing Criteria -- The basic purpose of any stage of testing determines
the criteria used to evaluate the test results. The testing criteria, in turn,
are reflected in the tests to be performed and the test methods employed. Testing
criteria derive from one of three objectives: data acquisition, determination of
specification compliance, and evaluation of mission performance. In developmental
testing the intent is to acquire comprehensive information on the characteristics
of the item under test. Usually, no a-priori criteria are imposed for performance
acceptance or rejection. Functional testing, however, is primarily intended to
evaluate the performance of the test item against specific criteria -- that is,
for specification compliance. As previously indicated, operational testing is
primarily concerned with mission performance. While some specific, quantitative

requirements are imposed, test criteria for operational testing often are of a qualitative nature.

It should be recognized that the three states of testing; developmental, functional, and operational; are not mutually exclusive. That is, the differences are primarily ones of emphasis. For example, functional testing often produces data that result in a design change. Thus, functional testing often takes on some aspects of developmental testing. For that reason, it is necessary, in functional testing, to test to a depth sufficient to allow engineering analysis of the problem. A "go" or "no-go" answer is not sufficient. On the other hand, functional testing cannot ignore mission suitability in evaluating a new design. Compliance with published specifications is not sufficient if functional testing reveals an operational problem. Thus, while the following sections of this test will be concerned primarily with quantitative tests for specification compliance, it should be noted that functional testing should reflect mission requirements, including non-quantitative considerations when appropriate.

In general, functional testing is required when any one of the following circumstances applies:

     (1) a new system is introduced
     (2) an existing system is significantly modified
     (3) the mission of an existing system is significantly extended
     (4) an existing system is installed in a new aircraft with significantly different environment.
     (5) an existing installation is extensively modified

With a digital system, some functional testing should be performed for even seemingly minor software changes.

5.1.3 Test Regimes -- Functional airborne system tests are performed in the laboratory, in the aircraft on the ground, and in flight. For various reasons, testing is usually performed in that order. Tests performed on the bench in the laboratory are most convenient, quickest, cheapest, and safest. Flight tests are least convenient, take the longest time, are most costly, and present the greatest danger to personnel and equipment. They also are most susceptible to uncertainties in the weather and availability of equipment. For the above reasons, tests should be performed in the laboratory, before installation in the aircraft, when feasible. Tests that can only be performed installed in the aircraft should be performed on the ground when feasible. Flight tests should be performed only when necessary and only when laboratory and ground tests have reduced the uncertainties to the greatest extent possible. Of course, some tests can be performed only in flight; and, in any event, flight performance eventually must be evaluated.

Flight tests sometimes can be performed in a test-bed aircraft. Such an arrangement allows in-flight tests to be performed with instrumentation far more extensive than would be possible with the system installed in the aircraft for which it was intended. In addition, a test bed aircraft can be employed for which flight operations are more convenient, less hazardous, and less costly. Testing in a test bed aircraft, however, cannot satisfy all flight testing requirements. The performance characteristics of all airborne systems are, to some extent, susceptible to the environment of the installation. Other factors influenced by the vehicle are the electrical power, cooling, electromagnetic interference, vibration, acceleration, and other environmental effects. In a digital system, software interaction is an important area for evaluation.

5.3

An alternative to some flight testing is flight simulation testing. The most useful "simulations" incorporate actual flight hardware for the system under test, utilizing simulations only for generating external stimuli. Such a hybrid test simulation can, in fact, perform tests not possible in actual flight. Test "flights" can, for example, be re-run exactly, or with controlled modifications. The ability of a simulation to exactly duplicate test conditions is especially valuable in testing digital systems, where one-at-a-time modifications of the inputs are necessary to exercise the various logic branches of the software. Furthermore, real-time interrupts in a test simulation make possible the examination of internal system quantities not available in an actual flight situation.

5.4

5.2 The Nature of Integrated Airborne Systems Testing

As previously indicated, an airborne weapons system is, in fact, a system of

systems. For that reason, the evaluation of an integrated airborne system

entails three distinct types of testing:

      (1) Subsystem Tests
      (2) Subsystem Interface Tests
      (3) Integrated System Tests

Subsystem tests evaluate the performance of the individual subsystems, acting

independently, and are presented in the texts devoted to the individual sub-

systems. Interface tests examine the ability of the subsystems to communicate

and to perform in concert. Interface tests include both functional tests and

subsystem compatibility tests concerned with environmental effects. Integration

tests evaluate the performance of the integrated system as a whole, examining

those functions beyond the capabilities of the individual subsystems acting

independently.

As a result of the exceedingly rapid evolution of integrated airborne systems

and of their great complexity, very few "standard" techniques of testing have

been formulated. For that reason, test procedures must be "tailored" to each new

system. In order to devise "tailored" test procedures the test designer must be

familiar with: (1) the principles of operation of the system to be tested,

(2) the operational (mission) requirements imposed upon the system, (3) the

general methods of testing applicable to that type of system, and, (4) the

general principles of test and evaluation.

5.3 The Testing Process

The testing process involves the critical steps listed below and discussed in
the following paragraphs.

    (1) Determine System Specifications
    (2) Determine Test Requirements
    (3) Develop Detailed Test Plan
    (4) Arrange for Test Facilities, Materiel, and Manpower
    (5) Perform Tests
    (6) Record Data
    (7) Process Data
    (8) Analyze Data
    (9) Document Test Results

Determining System Specifications -- It might seem that a complete set of well-
defined specifications for the test article would be an integral part of the
documentation accompanying a request for the preparation of a test plan.
Generally, this is not the case. The specifications that normally accompany such
a request are those to which the system contractor is contractually committed.
These specifications, usually prepared by the contractor, are often vaguely
defined and otherwise minimal. There does exist, however, a well-defined, com-
prehensive set of specifications. These specifications, documented for internal
use by the contractor, are the specifications to which the contractor's personnel
designed and constructed the system. Only from these detailed performance
specifications can the test designer fully understand the nature and intended
capabilities of the system. Most contractors make these specifications available
to the test designer upon request. The test program should reflect these detailed
specifications, even though the contractor can be held accountable only to the
contractual specifications. A failure of the contractor to meet his own design
goals reveals an area requiring careful examination in evaluating system com-
pliance with the contractual specifications.

Determining Test Requirements -- As with the system specifications, the test requirements included in a request for the preparation of a test plan are usually quite general. For that reason, those test requirements are useful only as a general guide in developing a detailed test plan. Some of the missing information can be derived from the contractor's detailed system specifications. The remaining information needed must be drawn from an understanding of the principles of operation and the mission requirements applicable to the system to be tested.

Developing the Detailed Test Plan -- The development of a detailed test plan

is, by far, the most important step in the testing process. If an adequate

test plan is generated, the actual performance of the tests is reduced to the

execution of routine procedures. Furthermore, the documentation of the test

results is greatly facilitated by the implementation of a detailed test plan.

(The on-going analysis of the test results is not of a routine nature, however,

and may require modification of the test plan.) There is, of course, a limit

to the degree of detail that can be written into a test plan. Total detail

cannot be written into a test plan unless the test results are already known.

On the other hand, an efficient, comprehensive test program cannot be conducted

until a detailed test plan is available. These conflicting requirements create

a dilemma for the test planner. The best approach is that of including in the

initial test plan as much detail as is possible, and updating that plan as more

detail becomes evident. An invaluable source of detail for the initial test

plan is the experience gained in previous test programs of a nature similar to

that of the program being planned. Detailed initial planning is extremely

important. Even in a well-planned program, however, some mid-program modifi-

cation of the test plan is likely. The development of a detailed test plan is

further discussed in Section 5.4 of this text.

Arranging for Test Facilities, Materiel, and Manpower -- At least preliminary

arrangements must be made, early in the planning phase, for the required program

support. It is likely that limitations in available support will, to some ex-

text, constrain the test program. An adequately detailed test plan cannot be

formulated, therefore, until those limitations are known. Once again, a planning

dilemma exists. The availability of resources is a function of the time period

during which specific tests will be performed. On the other hand, the testing

schedule cannot be known until a detailed test plan is available.

Performing the Tests -- As previously noted, sufficiently detailed test planning greatly facilitates the actual performance of the tests. The benefits of test planning are realized, however, only if the test program is conducted in strict adherence with the plan. Any deviation from the plan should be the result of carefully considered engineering judgment based upon the results of the on-going tests.

Recording the Data -- The recording of data must be included in the detailed test plan as an integral part of the testing process. Both qualitative and quantitative data must be carefully recorded. Any notable event, regardless of apparent significance, should be included in the record. Preparations should be made to record every quantity of possible interest. In the testing of a complex system, analysis of the data often reveals the need for data previously thought to be unrequired. It is good practice to utilize the available data recording facilities to capacity. Data acquisition is further discussed in Section 5.5 of this text.

Processing the data -- The processing of data (data reduction) is employed to: (1) transform the data signal into a form more suitable for viewing, recording, or further processing; (2) derive (compute) quantities not directly observable; (3) improve the signal-to-noise ratio, and (4) remove discontinuities caused by digitization, sampling, or data dropouts. Data processing is further discussed in Section 5.5 of this text.

Analyzing the Data -- Data analysis can be divided into two categories: quick-look analysis and detailed analysis. Quick-look analysis is conducted during the testing and is intended to reveal the need for repeated tests or further

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tests. Detailed analysis is performed after the planned testing and is intended to provide a detailed evaluation of the system under test. The statistical nature of test data analysis is discussed in Section 5.6 of this text.

Documenting the Test Results -- It is the nature of the testing process that the only product is the documentation. For that reason, a test program is only as good as the test report. As previously noted, a well-conceived detailed test plan greatly facilitates the preparation of the test report. The test plan provides a detailed outline of the body of the report -- that is, the description of the test results. Furthermore, the process of generating a detailed test plan identifies the purpose of the testing (the introduction) and the major results of the testing (the summary).

5.4 Test Plan Development

5.4.1 Test (Experiment) Design -- Effective test design involves the considerations listed below and discussed in the following paragraphs.

(1) The test must have a well-defined, specific objective.
(2) The parameters to be measured must provide for an unambiguous evaluation of the factor to be evaluated.
(3) Any interfering or obscuring parameters must be controlled or measured.
(4) The test measurements must provide sufficient accuracy.
(5) The test data should allow statistical determination of the testing error dispersions.
(6) A sufficient number of measurements must be made for the desired degree of confidence in the test results.
(7) The test procedures must ensure random, unbiased sampling of the parameter values to be measured.

Test Objectives -- The test plan must be based upon specific, well-defined objectives. "Evaluation of performance" is neither specific nor well-defined. In general, the test plan should call for specific measurements that provide a quantitative measure of performance.

Measured Parameters -- The measured (and/or controlled) parameters must be sufficient to completely define the system characteristics of interest.

Interfering Parameters -- All significant interfering inputs to the system must be controlled (or measured). A sensitivity analysis is required to identify the significant inputs.

Measurement Accuracy -- The accuracy of the measurements must be sufficient to yield the system parameters to the accuracy desired. A "brute force" approach is to perform the individual measurements to an accuracy much greater than that required for the parameters of interest. A better approach is to employ

5.11

statistics. If redundant measurements are performed, statistical data

processing (ranging from the simple arithmetic averaging of the measurements

to the use of a Kalman filter) can yield the desired accuracy, employing

individual measurements of an accuracy no greater than that desired for the

system parameter.

Test Result Statistics -- Quantitative test results should include some

statistical measure of the probability of error in the measured values. For

a normal (Gaussian) distribution of measurements, the mean and standard

deviation constitute the required measurement statistics. The mean is the

best estimate of the measured value. The standard deviation is a measure of

the dispersion (variability) in the measurements. The test measurements must

provide sufficient information to allow statistical analysis of the results.

Confidence Level -- In order to be meaningful, quantitative test results must

include at least two levels of statistical information. The means and standard

deviations constitute the first level (for Gaussian statistics). The second

level of statistics is expressed as a confidence level (or confidence interval).

The confidence level is a measure of the probable error in the first-level

statistics and, thus constitutes, in effect, statistics on the statistics. A

discussion of both first- and second-level statistics is contained in Section

5.6 of this text.

Random, Unbiased Measurements -- In order for the mean of the test measurements

to represent a true measure of the actual measured parameter, the measurements

must be unbiased -- that is, they must contain no errors with non-zero mean.

Furthermore, the systems upon which the measurements are made must be an unbiased

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sampling of the total population of such systems. Proper calibration of the test equipment and careful measurements satisfy the first requirement. Careful selection of the test samples satisfies the second. Often, the test designer has no control over the selection of test samples.

5.4.2 Detailed Test Planning -- The process of detailed test planning involves
the steps listed below and discussed in the following paragraphs.

        Identify Test Articles(s)
        Identify Tests Required
        Identify Test Program Constraints
        Formulate Detailed Test Procedure
        Document Test Plan
        Obtain Concurrence on Plan
        Obtain Approval of Plan

Identifying the Test Articles -- The first step in generating a test plan is
identification of the test article(s). An essential part of such identification
is the acquisition of a comprehensive set of specifications. (The problems
associated with obtaining detailed specifications are discussed in Section 5.3
of this text.) Such specifications, however, do not constitute a complete
identification of the test articles. Complete identification requires
identification of the actual hardware (by serial number) and verification that
the test articles actually represent unbiased, representative samples of the
system to be tested. Furthermore, on-going configuration control is required
to ensure that no unspecified adjustments or modifications are made, during
the test program, to the systems under test. It is also necessary, early in
the test planning stage, to determine the number of test articles to be made
available, the time periods of their availability, and their specified opera-
tional status.

Identifying the Tests Required -- The second step is generating a test plan
is identification of the test requirements. As previously indicated, the re-
quirements specified in the request for the preparation of the test plan
usually provide only general guidance. As indicated in Section 5.1.1 of this
text, the evolutionary status of the system determines the general nature of

the required testing (developmental, functional, or operational). The detailed testing requirements are derived from the detailed specifications of the system, a knowledge of the mission, and prior experience with similar systems.

The testing requirements are influenced also by any special requirements, with respect to test program results and documentation, imposed by the cognizant program office. Such special requirements should be identified early in the planning effort.

Identifying the Test Program Constraints -- The principal constraints generally imposed upon a test program are those listed below. Because a detailed test plan cannot be generated until all constrains are known, all such constraints should be identified before the detailed planning process is initiated.

> Time and Schedule Constraints
> Test Samples Available
> Testing Limits Imposed
> Facilities Available
> Manpower Available
> Materiel Available
> Funding Provided

Formulating the Detailed Test Procedures -- In addition to the considerations discussed in the foregoing paragraphs, the factors listed below must be included in the planning of detailed test procedures.

> Parameters to be Measured
> Parameters to be Controlled
> Required Accuracy of Measurements
> Required Number of Measurements
> Method of Testing to be Employed
> Required Data Acquisition and Processing
> Facilities and Materiel to be Utilized
> Manpower to be Utilized
> Testing Schedule Planned

The parameters to be measured and controlled are determined by the tests to be performed and the functional relationships between the parameters (mathematical model) of the system to be tested. Methods of data acquisition are discussed in Section 5.5 of this test. The measurement accuracies and the number of measurements required are determined by the mathematical model of the system and the data reduction to be employed. The statistical aspects of data reduction and analysis are discussed in Section 5.6 of this text. As previously indicated, an effort should be made to maximize the detail included in the test plan at every point in time. For that reason, the factors listed above should be established at the earliest opportunity and updated continuously.

Documenting the Test Plan -- Review, approval, and implementation of the proposed test plan require detailed documentation.

Obtaining Concurrence on Test Plan -- In order to avoid costly, difficult, and schedule-disrupting test plan alterations, a review of, and concurrence on, the proposed test plan should be sought from every organization involved in the test program.

Obtaining Approval of the Test Plan -- The detailed test plan should be submitted, at the earliest possible time, for the approval of both internal and external program managers.

5.5 Data Acquisition and Processing

5.5.1 Generalized Data Acquisition System -- The block diagram of a
generalized data acquisition system is shown in Figure 5.5.1.1. The sensor
or transducer generates a signal, (usually electrical), representing the
quantity to be measured. Typical sensors generate signals representing the
quantities listed below.

      Airspeed
      Pressure
      Temperature
      Voltage and Current
      Frequency and Phase
      Rotational Motion
      Linear Motion
      Mechanical Force and Strain
      Radiation Intensity
      Sound Level
      Time Interval

A signal conditioner modifies the signal in a manner that facilitates further
processing. Signal conditioners improve the signal-to-noise ratio, change
the voltage or power level of the signal, change the dynamic range or fre-
quency content of the signals, or change the modulation of the signal.
Typical signal conditioners are listed below.

      Amplifier/Attenuator
      Amplitude Limiter
      Frequency Filter
      Modulator (AM, FM, PM, PCM)
      Demodulator
      Frequency Converter
      Voltage-Controlled Oscillator
      Frequency Discriminator
      Digital Converter (A/D and D/A)
      Intervalometer
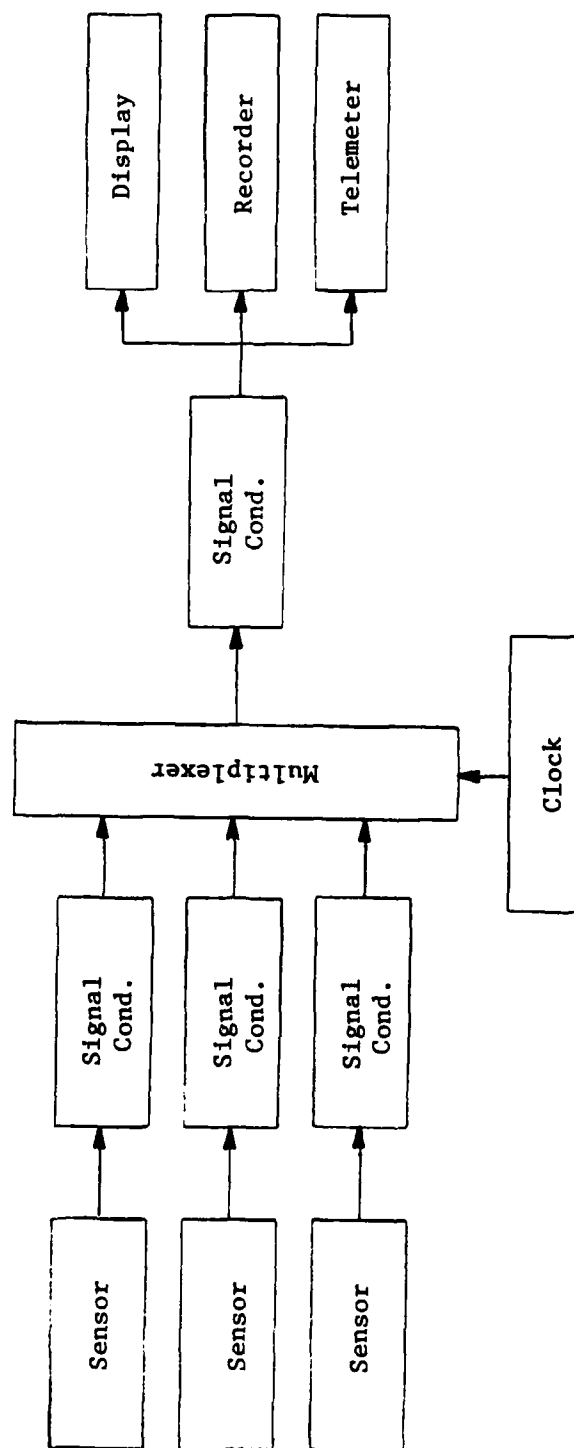   Time Sampler

Figure 5.5.1.1—Generalized Data Acquisition and Processing System

5.17a

A multiplexer is a device that allows a single data processing channel to carry two or more data signals. The individual signals are segregated by separating them in either the time domain or the frequency domain. That is, they share the single channel by time sharing (time division multiplex) or spectrum sharing (frequency division multiplex). Signal multiplexing is discussed in Section 5.5.5 of this text.

In order to time correlate recorded data, a time-base generator is required. The function of the clock shown in Figure 5.5.1.1 is to "time tag" the recorded data.

A telemeter is a device that transmits a signal from one location to another. It may consist of a simple transmission line or it may involve the transmission of radiant energy. The purpose of telemetry is to allow real-time processing, display, or recording, at a remote location. Techniques of data telemetry are discussed in Sections 5.5.3 and 5.5.4 of this text.

As indicated in Figure 5.5.1.1, a data signal may be displayed locally, recorded locally, or telemetered to a remote location. Real-time display is utilized when it is necessary to review the data as it is generated. A typical requirement for real-time display is the monitoring of parameters influencing flight safety. Most data are recorded in order to allow further processing and detailed analysis.

5.5.2 Data Acquisition System Characteristics -- The data acquisition system characteristics required for the measurement of a given system parameter are listed below.

> Accuracy
> Redundancy
> Bandwidth
> Dynamic Range
> Sampling Rate
> Time Labeling
> System Loading
> Output Format

The accuracy required in the individual measurements of a given parameter is determined by the accuracy required in the final determination of that parameter, the statistics of the measurement error, and the statistical data processing to be employed. The redundancy required in the measurement of a given parameter (number of individual measurements required) is determined by the confidence level required in the final determination of that parameter, the statistics of the measured parameter, the statistics of the measurement error, and the statistical data processing to be employed. The statistical nature of the measurement process, including statistical data processing and the resulting confidence level, is discussed in Section 5.6 of this text.

The required bandwidth of the measurement system is determined by the baseline bandwidth of the measured parameter and the sampling or data rate of the measurements.

The required dynamic range of the measurement system must be greater than that of the quantity to be measured.

The sampling rate of a sampled-data measurement system must be at least twice (preferably five times) the highest frequency present in the measured quantity.

The accuracy and resolution of the time labeling applied to the measurement of a given quantity is determined by the time correlation required in the data processing and analysis to be employed in data reduction.

In general, a measurement system must be designed to avoid significant system loading (the effect of the measuring system on the quantity to be measured). In some cases, the effects of unavoidable system loading can be determined and removed from the test results.

The measurement system must be designed to yield the test results in the format desired.

5.5.3 Frequency Modulation Data Processing -- A generalized analog FM data acquisition channel is shown in Figure 5.5.3.1. The accuracy of such a system is typically 2 percent of full scale. The data signal frequency passband typically extends from 10 hertz to 10 kilohertz. The principal advantages of an analog FM data system are the relatively wide bandwidth (compared to PCM), good noise rejection (compared to AM), and, in the absence of time multiplexing, continuous data capability.
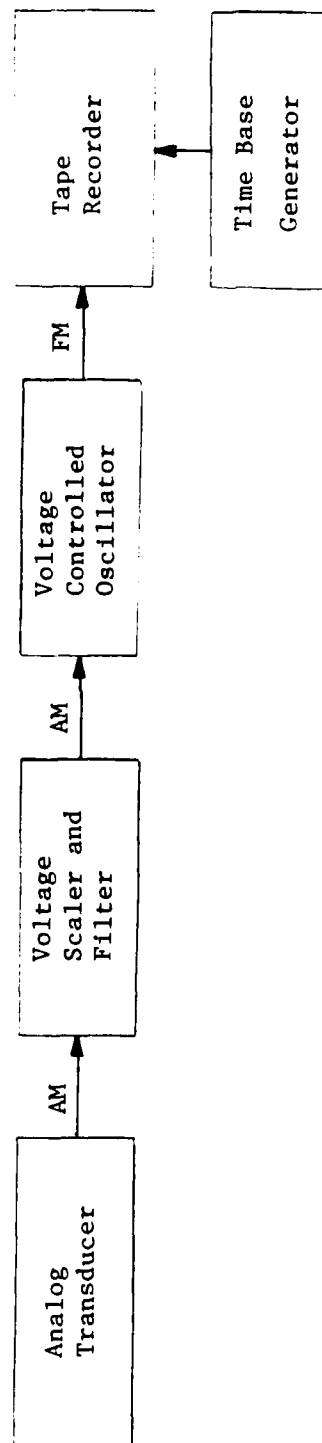
Figure 5.5.3.1 -- Analog Frequency Modulation Data Acquisition Channel

5.5.4 Pulse Code Modulation Data Processing -- A generalized PCM (digital) data acquisition channel is shown in Figure 5.5.4.1. The accuracy of such a system is typically 0.2 percent of full scale. The data signal frequency pass-band typically extends from zero to 100 hertz. The principal advantages of a PCM data system are high accuracy (compared to analog FM) and excellent noise rejection. The principal disadvantages of PCM are the sampled-data nature of the system and the small baseband data signal bandwidth (for a given PCM data rate). As a demonstration of the extremely high data rate produced by a relatively modest data telemetry system, consider the PCM system shown in Figure 5.5.4.2. Assume that the total number of measurements is 100, the re-quired sampling rate for each measurement is 1000 samples per second, and the number of bits per PCM word is 10 (0.5% accuracy with sign and parity). The PCM data (bit) rate is given by:

$$(\text{Data Rate}) = \begin{pmatrix} \text{Number of} \\ \text{Measurements} \end{pmatrix} \times \begin{pmatrix} \text{Sampling} \\ \text{Rate} \end{pmatrix} \times \begin{pmatrix} \text{Bits/Word} \end{pmatrix}$$

or:

$$(\text{Data Rate}) = 100 \times 1000 \times 10 = 10^6 \text{ Bits/Second}$$
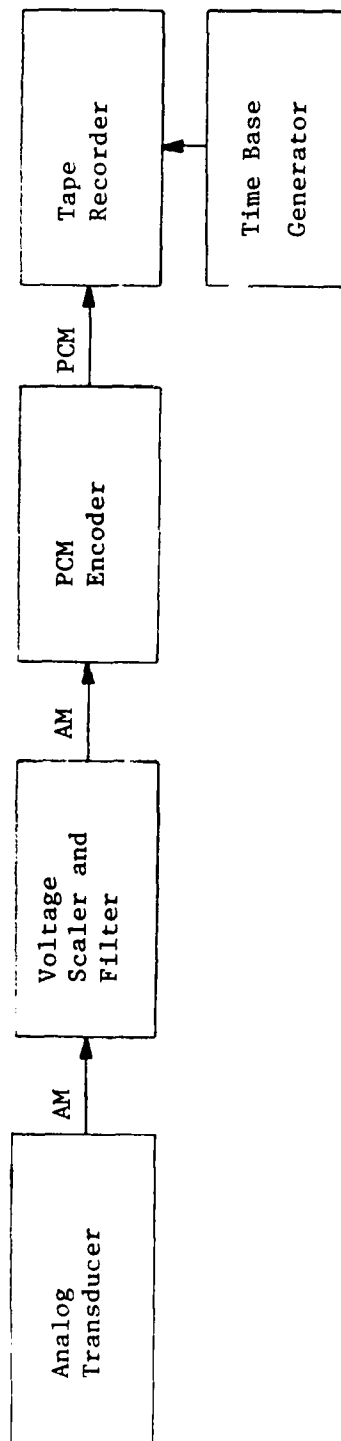
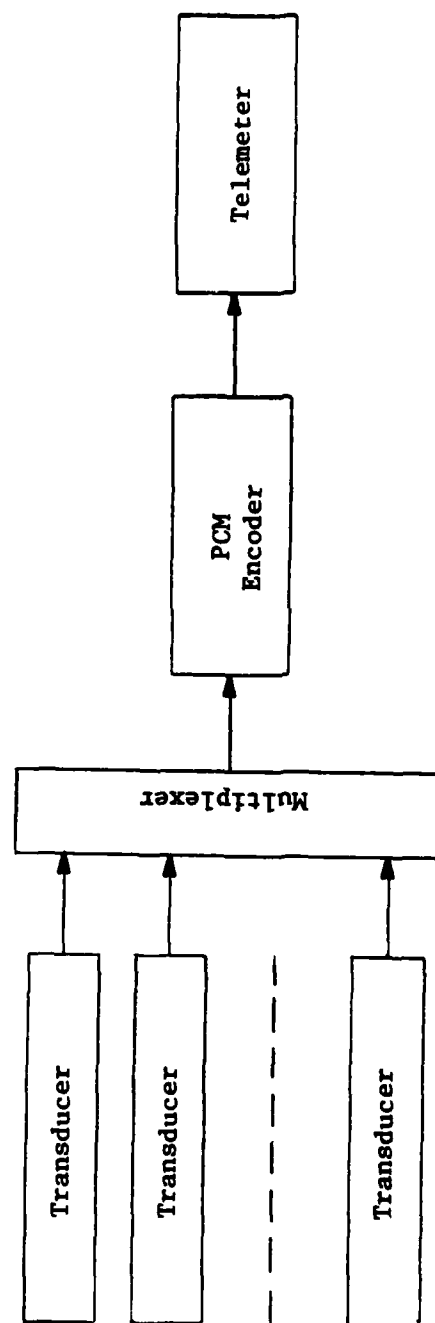Figure 5.5.4.1 -- Pulse Code Modulation Data Acquisition Channel

Figure 5.5.4.2—Typical PCM Data Acquisition System

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5.5.5 Data Multiplexing -- As previously indicated, multiplexing allows the simultaneous processing of two or more data signals by separating them in time or in frequency content.

An example of a time division multiplexer is the mechanical commutator shown in Figure 5.5.5.1. As indicated in the figure, any given signal can be sampled more than once per revolution of the commutator by cross-barring (electrically connecting two or more of the commutator segments). Also shown in the figure are two voltage reference calibration sources and two blank commutator segments used for frame synchronization. De-commutation is performed by a second "commutator" with the composite signal fed to the rotor and the individual signals taken from the commutator segments. Most modern signal commutators employ solid state, rather than mechanical, switching. In either case, the resulting time-division-multiplexed signal is similar to that shown in the figure.

The block diagram of a frequency division multiplexer is shown in Figure 5.5.5.2. As indicated in that figure, the amplitude modulated output of each transducer is input to a subcarrier oscillator consisting of a voltage-controlled oscillator. The frequency modulated output of each subcarrier oscillator is offset, in frequency, so as to separate its output spectrum from that of every other sub-carrier oscillator. The outputs of the subcarrier oscillators are then summed and the composite signal is used to modulate the carrier of the transmitter. De-multiplexing of the composite signal can be accomplished, as shown in part (b) of the figure, by passing the composite signal through a bank of band-pass filters, in parallel. In Figure 5.5.5.3 is shown a table of (proportional bandwidth) subcarrier band frequency allocations, their frequency deviation

(d) COMMUTATOR COMPOSITE OUTPUT

output for one revolution (one frame)

full-scale pulse

zero-level pulse

sync (no pulse for two contacts)

(a) COMMUTATOR

sync

output

zero-level reference

full-scale reference

TRANSDUCERS

A

B

C

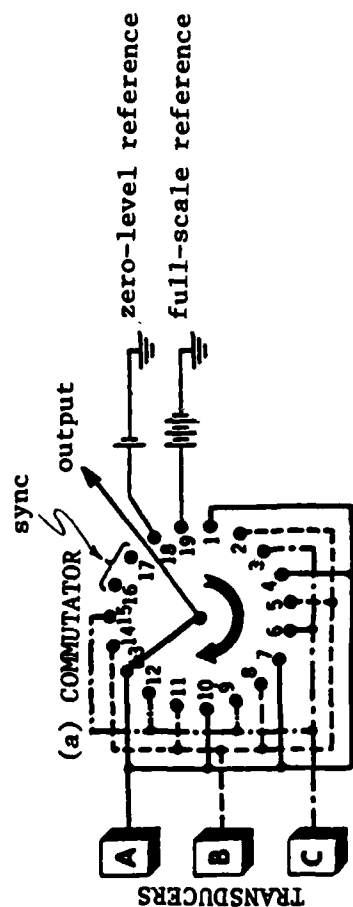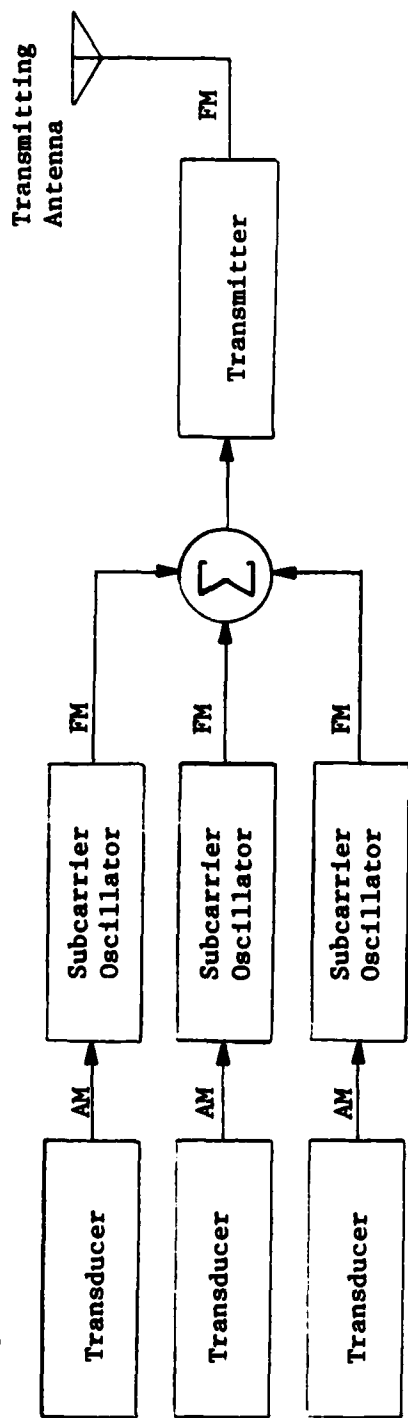(b) TRANSDUCER SIGNALS

(c) SAMPLED SIGNALS
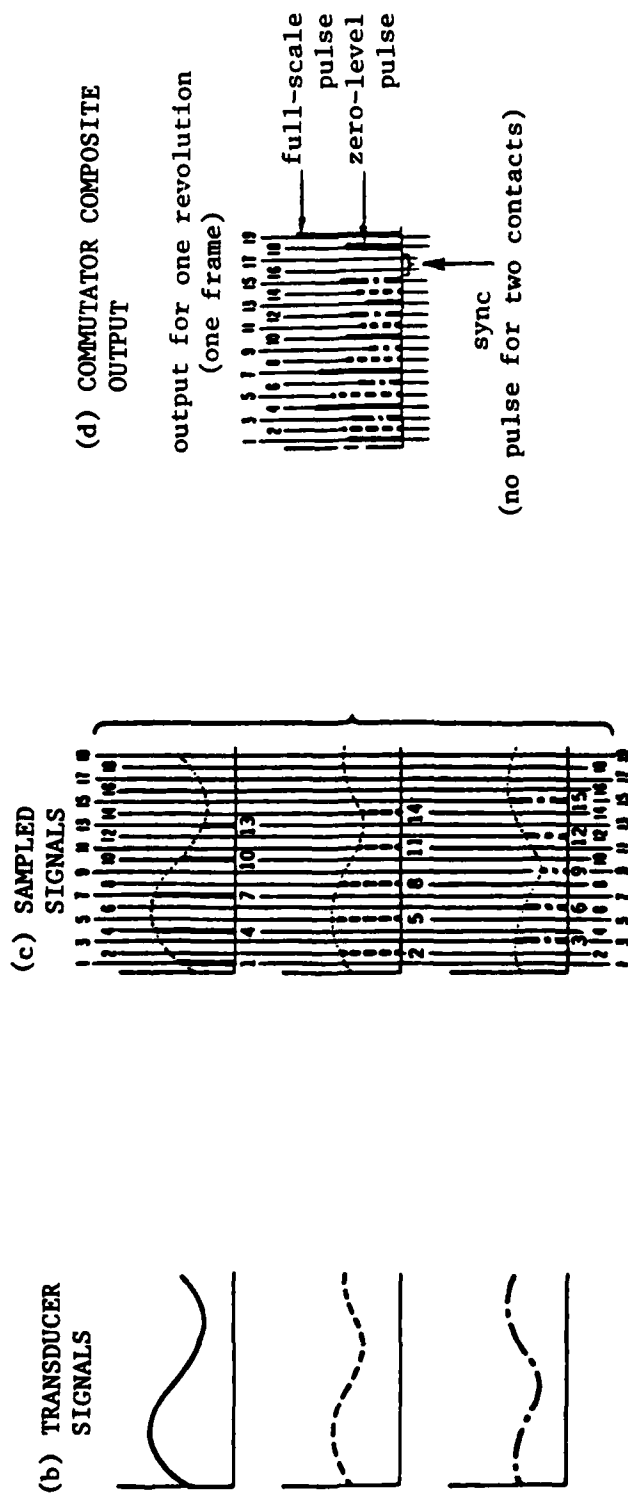
Figure 5.5.5.1—Time Division Multiplexing

5.23a

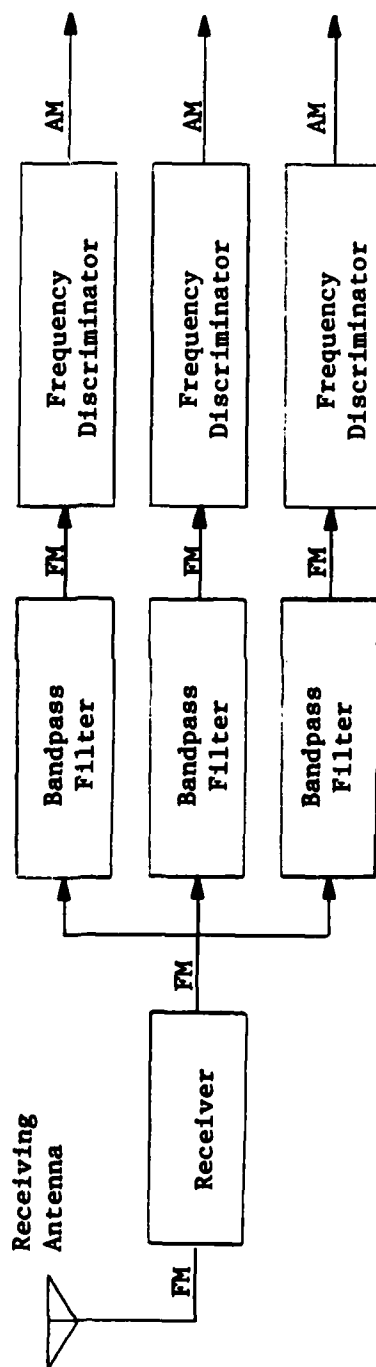**(a) Multiplexer**

(b) De-multiplexer

Figure 5.5.5.2--Frequency Division Multiplexing

5.23b

Band	Center Frequency (cps)	Lower Limit (cps)	Upper Limit (cps)	Maximum Deviation (percent)	Bandwidth (cps)
1	400	370	430	±7.5	6.0
2	560	518	602	"	8.4
3	730	675	785	"	11.
4	960	888	1,032	"	14.
5	1,300	1,202	1,398	"	20.
6	1,700	1,572	1,828	"	25.
7	2,300	2,127	2,473	"	35.
8	3,000	2,775	3,225	"	45.
9	3,900	3,607	4,193	"	59.
10	5,400	4,995	5,805	"	81.
11	7,350	6,799	7,901	"	110.
12	10,500	9,712	11,288	"	160.
13	14,500	13,412	15,588	"	220.
14	22,000	20,350	23,650	"	330.
15	30,000	27,750	32,250	"	450.
16	40,000	37,000	43,000	"	600.
17	52,500	48,560	56,440	"	790.
18	70,000	64,750	75,250	"	1,050.
A **	22,000	18,700	25,800	±15	660.
B	30,000	25,500	34,500	"	900.
C	40,000	34,000	46,000	"	1,200.
D	52,500	44,620	60,380	"	1,600.
E	70,000	59,500	80,500	"	2,100.

* Bandwidth shown is for proportional bandwidth system and is based on maximum deviation and a deviation ratio of five.

* Bands A through E may be used by omitting other interfering bands.

Figure 5.5.5.3--Frequency Division Multiplexing Subcarrier Band Allocations

limits, and the resulting information signal bandwidths. In Figure 5.5.5.4 is shown a table identifying the nomenclature used to describe the commonly employed data telemetry signal modulation combinations, including those using a frequency modulated subcarrier for frequency division multiplexing.

With respect to information-carrying capacity, the important figure-of-merit for a signal processing system is its channel-bandwidth product (the product of the total number of channels and the bandwidth of each channel). That is, the information carrying capacity of a given telemetry system depends only upon the total bandwidth of the system and is independent of the means employed (time-division or frequency-division) to multiplex the signals.

Nomenclature	First 3 Letters Data Signal Modulation	Middle 2 Letters Subcarrier Modulation (When Subcarrier Oscillator is used)	Last 2 Letters Carrier Modulation
PAM-FM-FM	Pulse Amplitude Modulation	Frequency Modulation	Frequency Modulation
PAM-FM-PM	"	"	Phase Modulation
PAM-FM-AM	"	"	Amplitude Modulation
PAM-FM	"		Frequency Modulation
PAM-PM	"		Phase Modulation
PAM-AM	"		Amplitude Modulation
PDM-FM-FM	Pulse Duration Modulation	Frequency Modulation	Frequency Modulation
PDM-FM-PM	"	"	Phase Modulation
PDM-FM-AM	"	"	Amplitude Modulation
PDM-FM	"		Frequency Modulation
PDM-PM	"		Phase Modulation
PDM-AM	"		Amplitude Modulation

Figure 5.5.5.4 -- Telemetry Signal Pulse Modulation Nomenclature

5.5.6 Test Range Instrumentation -- "Range instrumentation" is a term used
to signify that portion of the total measuring, telemetering, and processing
equipment at a test facility employed to determine the position and motion of
objects involved in tests. The objects tracked may be test vehicles, target,
missiles, or devices used to calibrate measuring equipment, obtain data on the
air mass, or alter the test environment.

Commonly employed range instrumentation tracking systems include radio frequency
radars, photo-theodolites, laser trackers, inertial navigation systems, radio
navigation systems, and missile-impact "splash nets". Radio frequency radar
trackers are employed to obtain line-of-sight bearing (aximuth and elevation),
range, and range rate, as described in the text on radar systems. Typical,
currently-available radars provide maximum tracking ranges of about 300 nautical
miles. When target skin tracking is employed, individual radar tracking
accuracies vary from 0.5 to 2 milliradians in bearing and from 15 to 75 feet
in range. When the target is provided with a radar transponder beacon, typical
accuracies vary from 0.1 to 1 milliradian in bearing and from 6 to 30 feet in
range.

Photo-theodolites are telescopes with provision for precise manual or automatic
tracking and an integral motion picture camera for recording line-of-sight
bearing data only. The bearing accuracy of "real time" theodolite tracking
data is about 0.5 milliradians. When manual (post-test) film reading is em-
ployed, photo-theodolite bearing accuracy is about 0.1 milliradians.

Laser trackers employ optical-frequency radiation to track a target in a manner similar to that employed by a radio-frequency radar. The laser tracker, however, requires the use of a corner reflector on the tracked object. The maximum range of a typical laser tracker is about 20 nautical miles. The position accuracy of a laser tracker is about 0.1 milliradians in bearing and varies from 1 to 3 feet in range, depending upon range.

An on-board inertial navigation system is, at present, the best available source of aircraft attitude real-time data. (At short ranges, theodolite films can be used, in conjunction with a model of the test aircraft, to obtain manual post-test approximations of vehicle attitude.) An inertial navigation system also provides vehicle position and velocity data with no restriction as to maximum range. The measurement accuracies obtainable with a typical INS are about 1 nautical mile in position, 3 feet per second in velocity, and 0.5 milliradians in vehicle attitude.

Current radio navigation devices used for range instrumentation are DME-like systems. (Distance Measuring Equipment systems are discussed in the text on navigation systems.) Typical DME-like position accuracies are about 50 feet over ranges of several miles.

A missile-impact net is a device consisting of a number of acoustic sensors located in the vicinity of a fixed target on the earth's surface. By measuring the times of arrival of the missile impact disturbance at the various sensor locations, the missile impact location can be determined within an accuracy of a few feet.

The range instrumentation installed at the Naval Air Test Center is depicted in Figure 5.5.6.1. When precise tracking is required, the measurements from a number of sensors are combined to obtain a least-squares best estimate of the target position. The error contours for a best-estimate solution employing the five theodolites shown are depicted in Figure 5.5.6.2. The estimated error corresponding to each contour is shown in parentheses (5 feet for contour number 1).
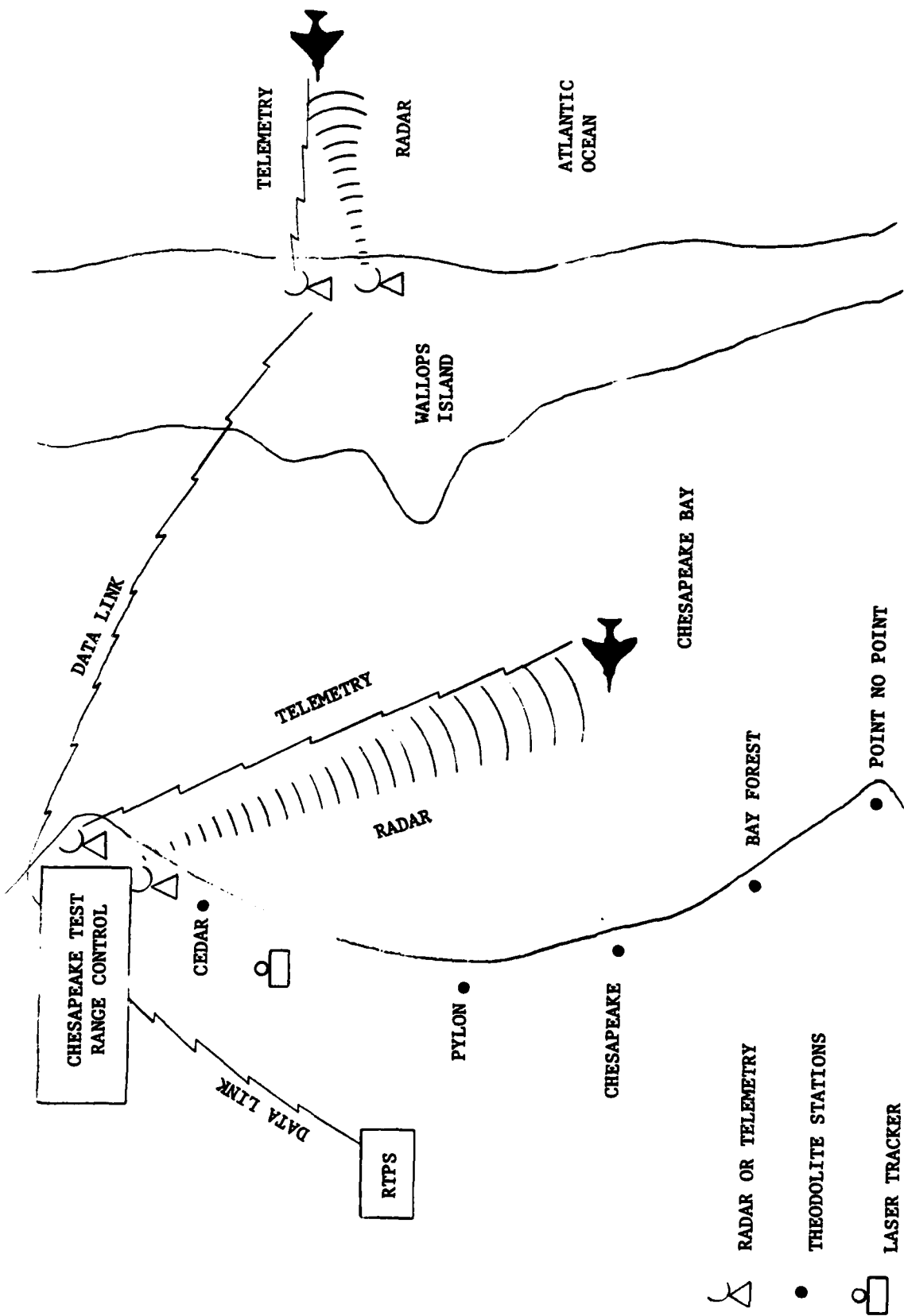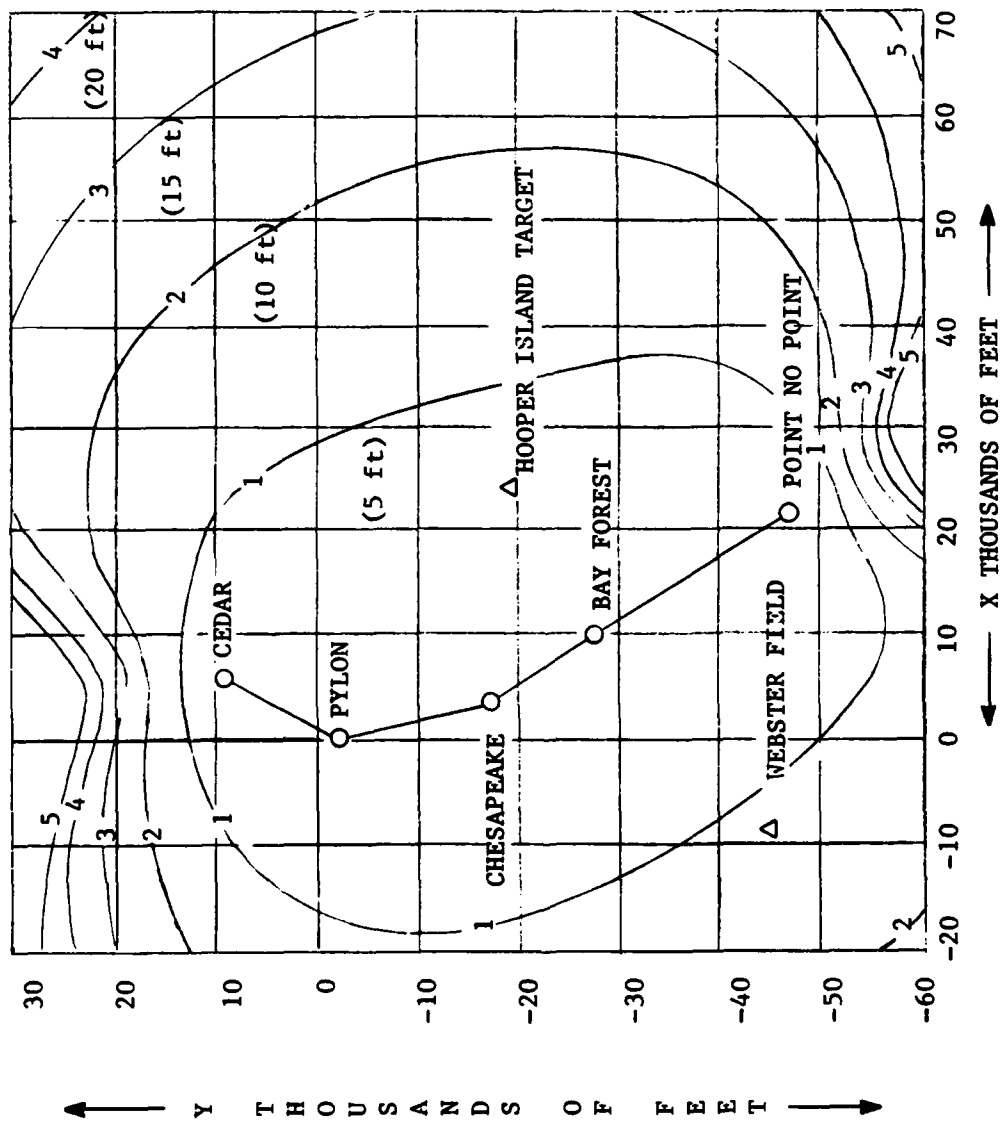
Figure 5.5.6.1—Range Instrumentation at NAVAIRTESTCEN

5.27a

Figure 5.5.6.2--Error Contours for Best Estimate Theodolite Tracking

5.27b

5.5.7 Real-Time Data Processing -- The real-time telemetry processing system (RTPS) installed at the Naval Air Test Center is an example of a modern, integrated, computerized data processing system. A block diagram of the RTPS is shown in Figure 5.5.7.1. As indicated in the figure, the data processing system receives telemetered data from the range or in-flight instrumentation, decodes and/or demodulates it, conditions it, computes derived system variables, and displays and records the results at the project engineers station. Both the raw (incoming) data and the processed data are recorded and may be re-played for later analysis. To expedite "real-time" analysis, the processed data are stored temporarily in random access memory and may be displayed immediately, (while still in temporary storage), for near-real-time analysis. Employing this technique, the project engineer can, for example, examine data time-expanded about a specific time point, such as that of weapon release. The recorded data also can be combined, for post-flight analysis, with data recorded in flight but not telemetered to RTPS in real-time, thereby allowing the utilization of data from otherwise inaccessible sources. The signal conditioning and data reduction functions performed by the RTPS are listed below.

Telemetering
Demodulation
Digital Decoding
Digitization
Linear Signal Conditioning
Nonlinear Signal Conditioning
Spectral Filtering
Time Synchronization
Measuring System Linearization
Measuring System Calibration
Interpolation (Smoothing)
Extrapolation (Prediction)
Curve Fitting
Limit Monitoring
Spectral Analysis
Statistical Analysis
Display
Recording

COMMUNICATION WITH PILOT

PROJECT ENGR STATION AND DISPLAYS

COMPUTATION PROGRAMS

1
2
3
4

REAL TIME COMPUTATION AND DISPLAY FORMATTING

RANGE INSTRUMENTATION DATA

SCALING, LIMIT, CHECK INFO

1
2
3
4

DATA CHANNEL

ENG. UNITS TAPE

SCALING, LIMIT, CHECK INFO

PCM FM

TIME

TELEMETRY DECODING SUBSYSTEM

ANALOG TAPE
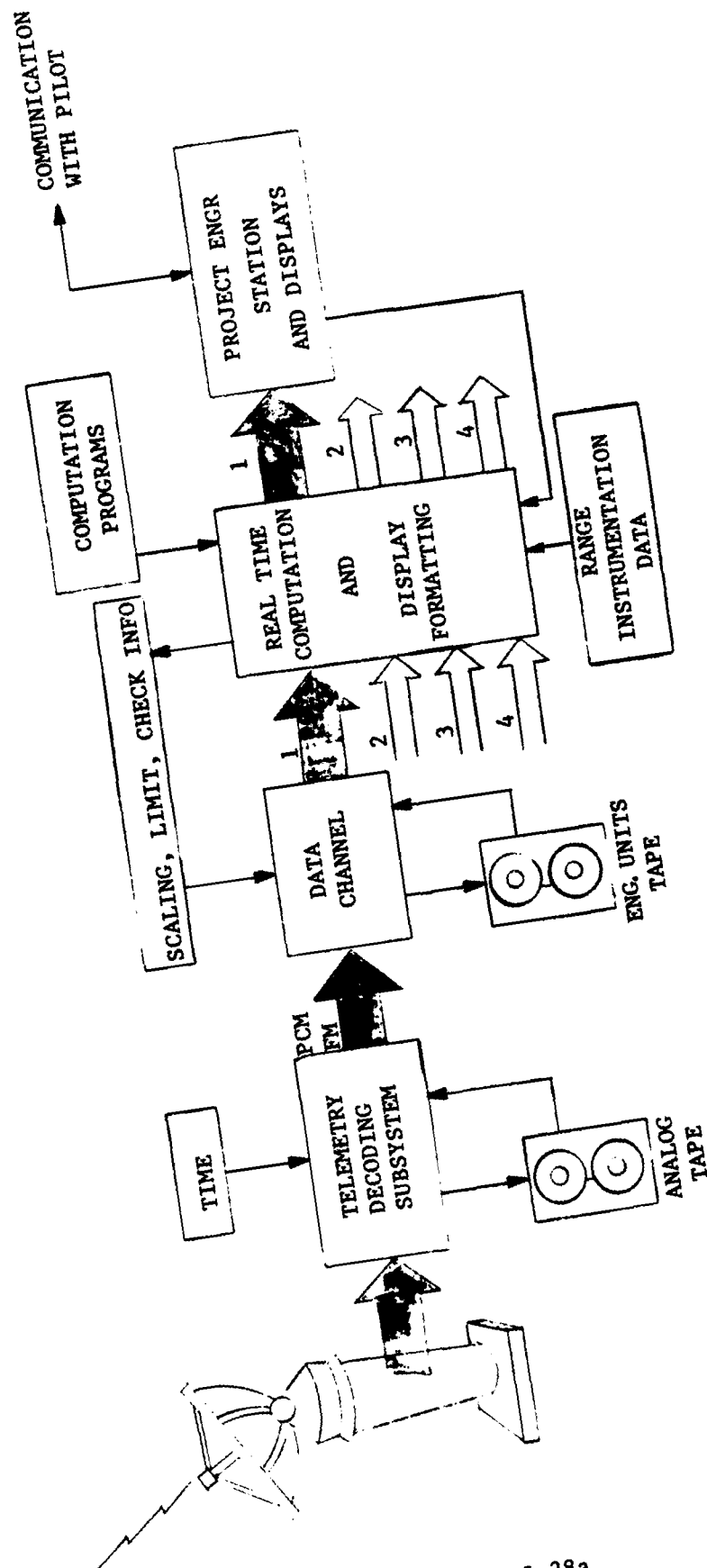
Figure 5.5.7.1--The Real-Time Telemetry Processing System

5.28a

The control center of the RTPS is the project engineer's station (PES). The instruments and controls at the PES allow the project engineer to pre-program the data reduction process, control the prosecution of the test, communicate with the other test personnel (including the flight crew), and examine displayed and recorded data. The information continuously available at the PES includes that listed below.

    Real-Time Time Plots
    Real-Time Cross Plots
    Selected Parameters
    Critical Parameters
    Out-of-Limit Parameters

5.6 Data Analysis

5.6.1 The Statistical Nature of Experimental Data -- It is the nature of the
testing process that both the systems under test and the measurement systems
themselves are stochastic. That is, they exhibit random signal components
and errors. (Random quantities are unpredictable; deterministic quantities
are predictable.) Thus, even if it were possible to eliminate all deterministic
errors, it still would be necessary to employ statistical methods in testing
and data analysis. Furthermore, test results never can be stated with absolute
certainty. Always, the value of a measured parameter must be stated in terms
of the probability that its true value lies within stated limits. In order to
reduce the uncertainty in the measured values of test parameters, multiple
(redundant) measurements must be made and the results statistically combined to
obtain a "best estimate" of the true value. The manner in which the test measure-
ments are combined, the resulting probability of error, and the level of con-
fidence in the results are determined by the statistical characteristics of
the testing process.

5.6.2 The Probability Density Function -- The statistical characteristics
of any process are identified by the applicable probability density function
(PDF). The PDF is best defined by its integral. That is, the integral, be-
tween given limits, of the PDF of a measurement with random error (the area
under the PDF curve between those limits) is equal to the probability that
the value of any given measurement will fall within the given limits. That
is:

$$\mathcal{P}\{x_1 \leq x \leq x_2\} = \int_{x_1}^{x_2} p(x)\, dx$$

where $\mathcal{P}\{x_1 \leq x \leq x_2\}$ is the probability that x will fall
between x_1 and x_2 and p(x) is the PDF of x. Note that p(x) is normalized so
that the total area under the PDF curve must be equal to unity. (The value of
x must fall somewhere ur J r the curve.)

For many measurement processes, the statistics are Gaussian (Normal). That is,
the PDF is given by:

$$p(x) = \left(\frac{1}{\sqrt{2\pi}\, \sigma_x}\right) e^{-\frac{(x-m_x)^2}{2\sigma_x^2}}.$$

where m_x is the mean of x and σ_x is the standard deviation of x. (Even non-
Gaussian processes yield Gaussian results when the number of individual
measurements is large and the "results" are obtained by taking the mean of
the individual measurements.) The Gaussian probability density function is
shown in Figure 5.6.2.1 with the mean and standard deviation indicated. For a
Gaussian distribution, the most frequently occurring value of the population

5.31

$$p(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_x}\right) e^{-\frac{(x - m_x)^2}{2\sigma_x^2}}$$
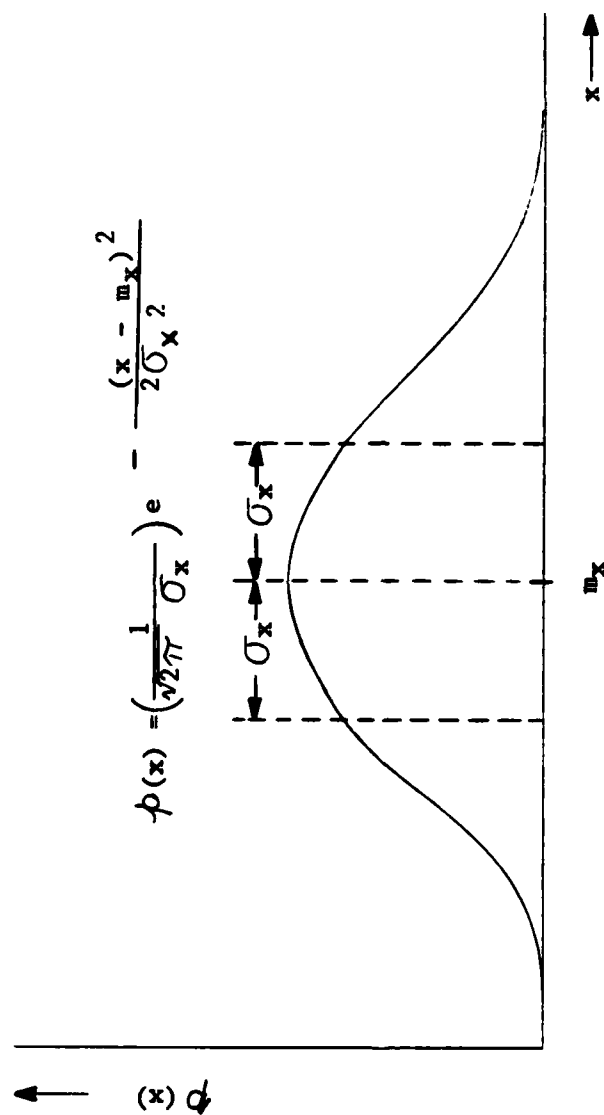
Figure 5.6.2.1—The Gaussian or Normal Probability Density Function

is the mean and about 68 percent of the total population fall within one standard deviation of the mean. Note that, for a Gaussian distribution, m_x and σ_x completely define the PDF.

5.6.3 Sample Statistics -- The determination of the true statistics of a
population, (mean and standard deviation for a Gaussian population), requires
measurements on every member of that population. In addition, random errors
in the measurements necessitate an infinite number of measurements on each
sample if the population statistics are to be determined with no uncertainty.
Generally, it is not feasible to test an entire population, nor is it possible
to perform an infinite number of measurements. In fact, the tester may have
access to only one or two samples of the population and the number of measure-
ments may be severely limited. When such is the case, the statistics de-
termined by testing are the sample statistics. If a reasonable number of
measurements are taken, the sample statistics may approximate the population
statistics. The interpretation and application of test results thus entails
an extrapolation of the results of measurements on a few samples (sample
statistics) to represent the characteristics of an entire population. The
validity of such an extrapolation requires an adequate number of unbiased
measurements on an adequate number of representative samples of the population.

The sample mean, $\bar{x}$, of a number of measurements is given by:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

where

N = Number of Measurements

x_i = Individual Measurements

The sample mean, $\bar{x}$, is an estimator of the true population mean, m_x.

5.33

The sample standard deviation, S_x, of a number of measurements is given by:

$$S_x = \left[\left(\frac{1}{N-1} \right) \sum_{i=1}^{N} (x_i - \bar{x})^2 \right]^{1/2}$$

The sample standard deviation, S_x, is an estimator of the true population standard deviation, σ_x.

5.6.4 The Confidence Interval -- As previously stated, the results of
measurements involving random variations or errors must be stated in
statistical terms. Generally, the sample mean of the measurements is quoted
as the best estimate of the value of the measured parameter. The sample
standard deviation is quoted as a measure of the uncertainty in the value of
the measured parameter. It is possible to calculate the sample mean and
sample standard deviation, according to the equations in Section 5.6.3 of
this text, utilizing only two measurements. Clearly, such a calculation
would yield the sample statistics to a very low level of confidence. A high
level of confidence requires a correspondingly high number of measurements.

The level of confidence is defined as the probability that the sample statistic,
($\bar{x}$ or S_x) , falls within a specified interval known as the confidence interval.
In the following development, the mathematical relationships between the con-
fidence level, confidence interval, sample statistics, and population statistics
are derived.

Confidence Interval for the Sample Mean -- For a normally distributed random
variate, x, with population mean m_x, and standard deviation σ_x, the sample
means, $\bar{x}$, based upon N independent measurements, will be normally distributed
with a population mean, $m_{\bar{x}}$, and population standard deviation, $\sigma_{\bar{x}}$, given by:

$$m_{\bar{x}} = m_x$$

and:

$$\sigma_{\bar{x}} = \left(\frac{1}{\sqrt{N}}\right) \sigma_x$$

5.35

The probability, $\mathscr{P}\{\ \}$, that any calculated value of $\bar{x}$ will lie between the values $(1-r)\ m_x$ and $(1+r)\ m_x$ is then given by:

$$\mathscr{P}\left\{(1-r)\,m_x \leq \bar{x} \leq (1+r)\,m_x\right\} = 2 \int_{m_x}^{(1+r)m_x} p(\bar{x})\, d\bar{x} = P$$

where $2\,r\,m_x$ is the confidence interval of interest. Since $\bar{x}$ is normally distributed, the right side of the above equation can be evaluated by reference to tabulated values of the integral of the Gaussian probability density function. Values of the integral of the Gaussian PDF are tabulated for random variables of zero mean and unity standard deviation. In order to put the probability integral in standard (tabulated) form, the normalized variate y is substituted for $\bar{x}$ where:

$$y = \frac{\bar{x} - m_{\bar{x}}}{\sigma_{\bar{x}}}$$

or

$$\bar{x} = \sigma_{\bar{x}}\, y + m_{\bar{x}}$$

The probability equation then becomes:

$$\mathscr{P}\left\{(1-r)\,m_x \leq \bar{x} \leq (1+r)\,m_x\right\} = 2 \int_{0}^{y_1} p(y)\, dy = P$$

where:

$$y_1 = \frac{r\,m_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{r\,\sqrt{N}\,m_x}{\sigma_x} \cong \frac{r\,\sqrt{N}\,\bar{x}}{S_x}$$

The integral on the right side of the above equation can now be taken directly from the standard table of Gaussian integrals. Employing the tabulated values in conjunction with the above equations, the following determinations can be made.

5.36

(1) Given the statistics for the sample mean, ($m_{\bar{x}}$ and $\sigma_{\bar{x}}$), or estimates of the population, (m_x and σ_x), the number of measurements, (N), and the required confidence interval, ($2rm_x$), find the probability, (P), that a given calculated value of $\bar{x}$ will fall in the interval between $(1-r)\, m_x$ and $(1+r)\, m_x$.

(2) Given the statistics for the sample mean, ($m_{\bar{x}}$ and $\sigma_{\bar{x}}$), or estimates of the statistics for the population, (m_x and σ_x), the number of measurements, (N), and the required level of confidence, (P), find the confidence interval, ($2rm_x$), over which the required confidence level is satisfied. The confidence interval parameter, (r), is related to the integration upper limit, (y_1), by the equation:

$$r = \frac{\sigma_{\bar{x}}\, y_1}{m_{\bar{x}}} \cong \frac{\sigma_x\, y_1}{\sqrt{N}\; m_x}$$

(3) Given estimates of the statistics for the population, (m_x and σ_x), the required confidence level, (P), and the required confidence interval, ($2rm_x$), find the minimum number of measurements, (N), required to provide the given confidence level over the given confidence interval. The required number of measurements, N, is related to the upper integration limit, (y_1), by the equation:

$$N = \left[\frac{\sigma_x}{r} \frac{y_1}{m_x} \right]^2$$

The preceding discussion has been concerned with the probability that the sample mean will fall between two limits stated in terms of the true population mean; that is, with the probability:

$$P\left\{ (1-r)\, m_x \leq \bar{x} \leq (1+r)\, m_x \right\}$$

5.37

Also of interest is the probability that the true value of the population mean will fall between two limits stated in terms of the measured sample mean. In this respect, it should be noted that the above two statements of probability are uniquely related. That is:

$$\mathscr{P}\left\{(1-r)m_x \leqq \bar{x} \leqq (1+r)m_x\right\} \equiv \mathscr{P}\left\{\left(\frac{\bar{x}}{1+r}\right) \leqq m_x \leqq \left(\frac{\bar{x}}{1-r}\right)\right\}$$

The preceding methods of determination apply to requirements based upon either statement of probability.

Confidence Interval for the Sample Standard Deviation -- For a normally distributed random variate, x, with population mean m_x and population standard deviation σ_x, the sample variances, s_x^2, based upon N independent measurements, will not be normally distributed, but will exhibit a Chi-Squared, distribution with (N-1) degrees of freedom. Defining the normalized random variate:

$$\chi^2 = \frac{N s_x^2}{\sigma_x^2} \quad ,$$

The probability that s_x^2 will fall in the interval $\left(\chi_1^2 \leqq \chi^2 \leqq \chi_2^2\right)$ can be written:

$$\mathscr{P}\left\{\chi_1^2 \leqq \chi^2 \leqq \chi_2^2\right\} = \int_{-\infty}^{\chi_2^2} p(\chi^2)\, d\chi^2 - \int_{-\infty}^{\chi_1^2} p(\chi^2)\, d\chi^2 = P$$

5.38

or, in terms of S_x^2:

$$\mathscr{P}\left\{\left(\frac{\chi_1^2}{\mathcal{N}}\right)\sigma_x^2 \leq S_x^2 \leq \left(\frac{\chi_2^2}{\mathcal{N}}\right)\sigma_x^2\right\} = \int_{-\infty}^{\chi_2^2} p(x^2)dx^2 - \int_{-\infty}^{\chi_1^2} p(x^2)dx^2 = P$$

The basic principles involved in evaluating the right-hand side of the above equation are identical to those involved in the equation, previously considered, applicable to the sample mean, $\bar{x}$. The Chi-Squared distribution, however, requires three parameters for its specification. (The Gaussian distribution requires only two). The "extra" parameter is the number of degrees of freedom, ν, equal to one less than the number of measurements, N. For probability computations in which the number of measurements is known (or assumed), this extra parameter creates no difficulty. When, however, it is desired to compute the number of measurements required to attain a given level of confidence in the experimentally determined value of the sample variance, the value of ν is unknown. In such situations, an iterative procedure is required, in which a value is assumed for ν and then checked for consistency with the results of the computation. (The procedure is described later in this section.) Employing tabulated values of the Chi-Squared distribution integrals, and the above equations, the following determinations can be made.

(1) Given estimates of the statistics for the population, (m_x and σ_x), the number of degrees of freedom, ($\nu = N-1$), and the required confidence interval limits for the sample variance, $\left(\frac{\chi_1^2}{\mathcal{N}}\right)\sigma_x^2$ and $\left(\frac{\chi_2^2}{\mathcal{N}}\right)\sigma_x^2$, find the probability, (P), that a given calculated value of S_x^2 will fall in the specified confidence interval.

5.39

(2) Given estimates of the statistics for the population, (m_x and σ_x), the number of degrees of freedom, ($y = N-1$), and the required confidence level, (P), find the confidence interval limits, $\left(\frac{\chi_1^2}{\nu} \cdot \sigma_x^2 \text{ and } \frac{\chi_2^2}{\nu} \cdot \sigma_x^2 \right)$, over which the required confidence level is satisfied. Since the Chi-Squared distribution curve is not symmetric, the values of the two χ^2 limits, (χ_1^2 and χ_2^2), will be independent even if the assumption is made that the areas under the "tails" of the curve are equal (a common assumption). Thus, the two confidence interval limits, $\left(\chi_1^2 \text{ and } \chi_2^2 \right)$, must be determined separately, assuming an appropriate area (probability) under each "tail". That is, the probability equation must be expressed as two equations. Thus, the equation:

$$ \mathcal{P}\left\{ \left(\frac{\chi_1^2}{\nu} \right) \sigma_x^2 \leq S_x^2 \leq \left(\frac{\chi_2^2}{\nu} \right) \sigma_x^2 \right\} = \int_{-\infty}^{\chi_2^2} p(\chi^2)\,d\chi^2 - \int_{-\infty}^{\chi_1^2} p(\chi^2)\,d\chi^2 = P $$

is equivalent to the two equations:

$$ \mathcal{P}\left\{ S_x^2 \leq \left(\frac{\chi_1^2}{\nu} \right) \right\} = \int_{-\infty}^{\chi_1^2} p(\chi^2)\,d\chi^2 = P_1 $$

$$ \mathcal{P}\left\{ S_x^2 \leq \left(\frac{\chi_2^2}{\nu} \right) \right\} = \int_{-\infty}^{\chi_2^2} p(\chi^2)\,d\chi^2 = P_2 $$

where: $P_2 = P + P_1$.

If equal probabilities are assumed in the "tails" outside the interval:

$$ P_1 = \frac{1 - P}{2} \qquad \text{and} \qquad P_2 = \frac{1 + P}{2} $$

(3) Given estimates of the statistics for the population, (m_x and σ_x), the required confidence level, (P), and the required confidence interval limits, $\left(\frac{\chi_1^2}{N} \sigma_x^2 \text{ and } \frac{\chi_2^2}{N} \sigma_x^2 \right)$, find the minimum number of measurements, (N), required to provide the given confidence level over the given confidence interval. As in the preceding computation, the two confidence interval limits, $\left(\chi_1^2 \text{ and } \chi_2^2 \right)$, must be independently determined from the equations:

$$\mathscr{P}\left\{ S_x^2 \leq \frac{\chi_1^2}{N}\sigma_x^2 \right\} = \int_{-\infty}^{\chi_1^2} p(\chi^2)\, d\chi^2 = P_1$$

$$\mathscr{P}\left\{ S_x^2 \leq \frac{\chi_2^2}{N}\sigma_x^2 \right\} = \int_{-\infty}^{\chi_2^2} p(\chi^2)\, d\chi^2 = P_2$$

In general, the number of measurements, N_1, required to achieve the required confidence, P_1, in the lower limit, χ_1, is not equal to the number of measurements, N_2, required to achieve the required confidence, P_2, in the upper limit, χ_2. Therefore, both N_1 and N_2 must be determined and the minimum number of measurements required taken as the larger of the two. The procedure for determining either N_1 or N_2 is as follows.

(a) Assume a value for the number of measurements, (N_1 or N_2), or for the number of degrees of freedom, (ν_1 or ν_2), where ν = N-1.

(b) From tables of Chi-Squared distribution integrals, obtain the limit of integration, (χ_1^2 or χ_2^2), corresponding to the given probability, (P_1 or P_2), and to the assumed number of degrees of freedom, (ν_1 or ν_2).

(c) Denoting the value of the limit obtained in Step (b) as $(\chi^2)_{table}$, calculate a value for the required number of measurements from the equation:

$$\left(N \right)_{calculated} = \frac{(\chi^2)_{Table}}{\left(\frac{\chi^2}{N} \right)_{Given}}$$

where $\left(\dfrac{\chi^2}{N}\right)_{\text{Given}}$ is the value given for the required confidence interval limit.

(d) Compare the calculated value of N with the assumed value of N.

(e) Repeat steps (a) through (d) as required to obtain sufficient agreement between the calculated and assumed values of N.

Plots of the number of measurements required to determine the sample standard deviation to a given confidence level, (80%, 90%, and 95%), over a given confidence interval, (in percentage of S_x), are shown in Figure 5.6.4.1.

It should be noted that it is not necessary to assume equal probabilities below the lower limit, (P_1), and above the upper limit, $(1-P_2)$. These probabilities can, instead, be adjusted so as to result in a single, optimal required number of measurements, (N), to achieve both required probabilities. The curves in Figure 5.6.4.1 were generated employing such an adjustment. It also should be noted that the curves in Figure 5.6.4.1 are directly applicable to computations concerned with the circular error probable, (CEP), for weapon delivery data.
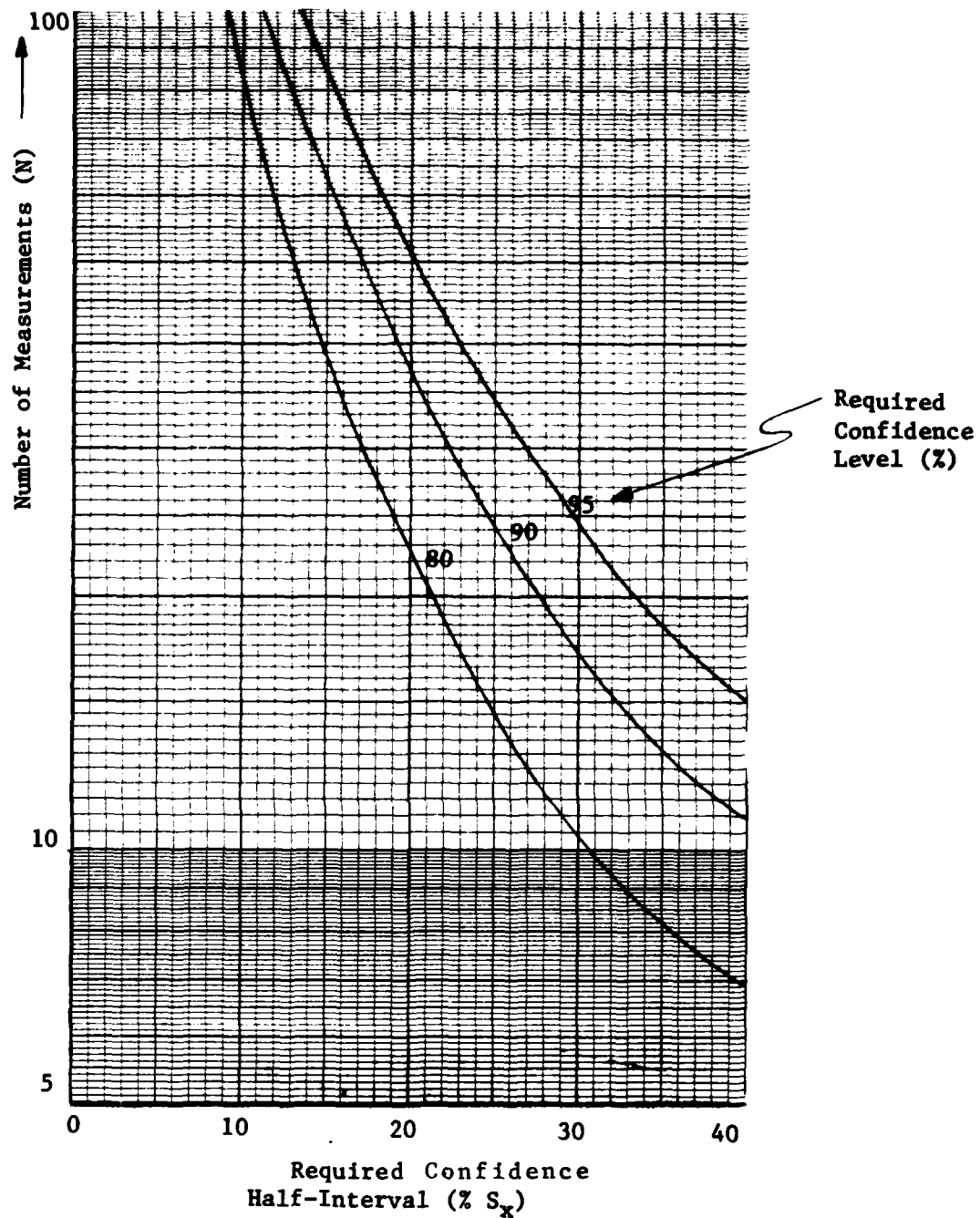
Figure 5.6.4.1--Number of Measurements Required to Estimate
Standard Deviation to a Required Accuracy
and Confidence Level

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5.6.5 Regression Analysis -- Regression analysis is a data reduction process by means of which the parameters of the functional relationship between two or more variables are derived by manipulation of correlated data produced by that functional relationship. An important example of regression analysis is linear, least-squares curve fitting; that is, the process of determining the parameters of the straight line that best fits a given set of data points. In least-squares curve fitting, the criterion for the "best fit" is minimization of the sum of the squares of the deviations of the given data points from the fitted line. Linear, least-squares curve fitting is illustrated in Figure 5.6.5.1. As indicated in that figure, the fitted line is represented by the equation:

$$y_i = a x_i + b$$

The deviation of the ith data point from that line is then:
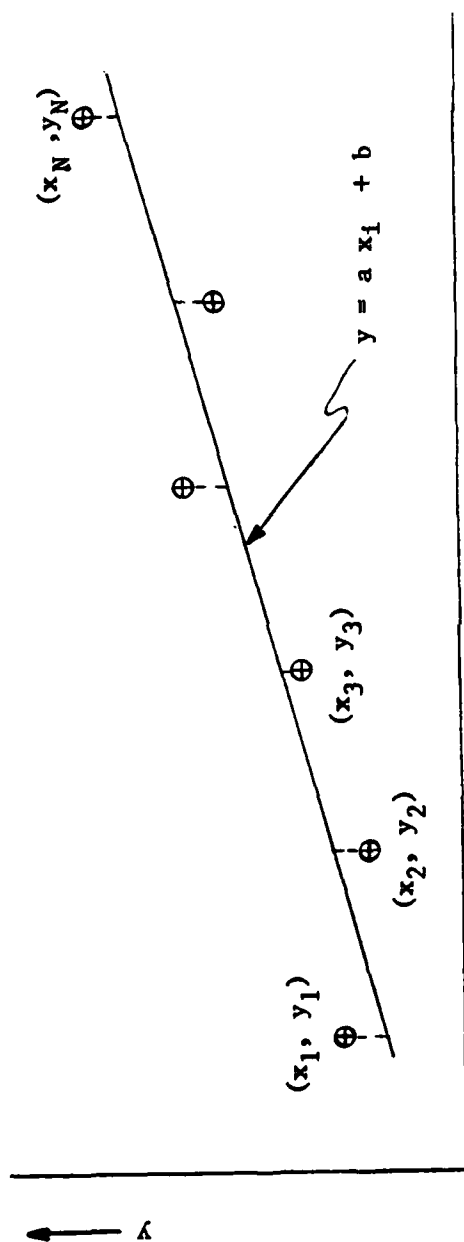
$$(y_i - a x_i - b)$$

and the sum of the squares of the deviations for all N data points is:

$$\sum_{i=1}^{N} (y_i - a x_i - b)^2$$

Differentiating the above cost function by the parameters a and b, setting the derivatives equal to zero, and solving the resulting equations for a and b yields:

$$a = \frac{\sum_{i=1}^{N} (x_i y_i) - N \bar{x} \bar{y}}{\sum_{i=1}^{N} x_i^2 - N (\bar{x})^2}$$

$$b = \bar{y} - a \bar{x}$$

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$$\text{Minimize} = \sum_{i=1}^{N} (y_i - ax_i - b)^2$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$$

$$a = \frac{\sum_{i=1}^{N} (x_i \, y_i) - N \, \bar{x} \, \bar{y}}{\sum_{i=1}^{N} (x_i^2) - N \, \bar{x}^2}$$

$$b = \bar{y} - a \bar{x}$$

Figure 5.6.5.1--Linear Least Squares Curve Fitting (Regression Analysis)

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where:

$$\bar{x} = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i$$

$$\bar{y} = \frac{1}{N} \cdot \sum_{i=1}^{N} y_i$$

The "best" straight line fit to the data points is then the line represented
by the equation:

$$y = a x + b$$

where a and b are defined by the above equations. It is important to note
that least-squares curve fitting can be applied to nonlinear relationships.
It also is important to note that "best fit" criteria other than least-squares
can be employed in curve fitting.

5.6.6 Sensitivity Analysis -- Sensitivity analysis is the process of determining the degree to which a system variable is sensitive to (affected by) changes in another system variable or test parameter. Sensitivity analysis is utilized in (pre-test) test planning to identify those parameters to which the test results are sensitive, and which must therefore be controlled and/or measured, and to determine the accuracy with which those parameters must be known. Sensitivity analysis is used in (post-test) data analysis to determine the contributions of the various error sources to system error (create an "error budget"), and to verify that all major sources of error have been included in the error model.

Basically, the sensitivity of one (dependent) variable to changes in another (independent) variable is determined by evaluating the first derivative of the dependent variable with respect to the independent variable. When the analytic relationship between the two variables is known, the sensitivity can be determined analytically by evaluating the analytic expression for the derivative. If experimental evaluation is feasible, the sensitivity can be determined empirically by measuring the change in the dependent variable due to a measured or controlled change in the independent variable, while holding all other independent variables constant.

Given the analytic functional relationship:

$$z = f(x, y)$$

the variations in z due to variations in x and y are given by:

$$(\delta z)_x = \left(\frac{\partial f}{\partial x}\right) \partial x$$

$$(\partial z)_y = \left(\frac{\partial f}{\partial y}\right) \partial y$$

The sensitivities of z to x and y are then:

$$S(z/x) = \left[\frac{\partial f}{\partial x}\right]_y$$

$$S(z/y) = \left[\frac{\partial f}{\partial y}\right]_x$$

When the sensitivities are determined empirically, the derivatives are replaced by differentials; that is:

$$S(z/x) = \left[\frac{\Delta z}{\Delta x}\right]_y$$

$$S(z/y) = \left[\frac{\Delta z}{\Delta y}\right]_x$$

Utilizing the partial derivatives derived from a sensitivity analysis, the total error (variation) in the dependent variable due to errors (variations) in two or more independent variables can be calculated. When the variations in the independent variables are deterministic, the total variation in the dependent variable is the sum of the contributions due to the individual independent variables. That is, for a dependent variable z and independent variables x and y related by the analytic expression:

$$z = f(x, y),$$

the total variation in z, Δz, is given by:

$$\Delta z = \left(\frac{\partial f}{\partial x}\right)\Delta x + \left(\frac{\partial f}{\partial y}\right)\Delta y$$

where Δx and Δy are the variations in x and y. When the variations in x and y are random and statistically independent, the contributions due to their variations must be RSS'd (root-sum-squared). That is:

$$\Delta z = \left[\left(\frac{\partial f}{\partial x}\right)^2(\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2(\Delta y)^2\right]^{1/2}$$

When the variations in x and y are random but not statistically independent, their correlation must be included in the total. That is:

$$\Delta z = \left[\left(\frac{\partial f}{\partial x}\right)^2(\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2(\Delta y)^2 + \rho_{xy}\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)(\Delta x)(\Delta y)\right]^{1/2}$$

where ρ_{xy} is the correlation coefficient for x and y. In terms of the variances (or standard deviations) of the random variables, the total variational equation can be written:

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2\sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2\sigma_y^2 + \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)\rho_{xy}\,\sigma_x\,\sigma_y$$

The correlation coefficient, ρ_{xy}, can be approximated, using test data, by the sample correlation coefficient given by:

$$R_{xy} = \left(\frac{1}{S_x\,S_y}\right)\left(\frac{1}{N-1}\right)\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$$

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5.6.7 The Kalman Filter -- As previously indicated, systems testing is a
stochastic process. That is, the measurements obtained are characterized by
random (unpredictable) errors. As discussed in Section 5.6.3 of this text,
statistical processing of redundant measurements is required in order to
reduce the effects of the random variations and obtain a "best estimate" of
the measured quantities. One method of obtaining a best estimate from the
redundant measurements is the calculation of the sample mean, as discussed in
Section 5.6.3. As defined therein, calculation of the sample mean is a "batch"
data processing technique; that is, one in which all of the redundant measure-
ments are simultaneously combined, (in a "batch"), to obtain a best estimate.
An alternative to batch data processing is sequential data processing,
(sequential estimation), in which continuing measurements are sequentially
incorporated into the best estimate, thereby improving the estimate with each
new measurement. Kalman filtering is such a sequential estimation process.
A Kalman filter combines continuing (redundant) data in such a way as to pro-
duce a statistically optimal (best) estimate of the variables and parameters
of the system under test. Like the (batch) regression analysis (curve
fitting) technique discussed in Section 5.6.3, a Kalman filter utilizes a
"least-squares" criterion to obtain a best estimate of the system parameters
and variables, by (sequentially) fitting time plots (curves) of the estimates
to the continuing measurement data.

In the calculation of the sample mean discussed in Section 5.6.3, each measure-
ment is weighted equally in obtaining the best estimate. In Kalman filtering,
the weighting factor applied to each measurement is inversely proportional to
the assumed variance of the error in that measurement and directly proportional
to a continually updated estimate of the variance of error in the estimated

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quantity. Thus, "poor" measurements are discounted; and, as the estimate improves, the weighting on all further measurements is progressively reduced.

Another feature of the Kalman filter is the incorporation of mathematical models of both the measurement process and the system under test. Utilizing these models, the Kalman filter is able to estimate quantities not directly measured. (The only requirement for estimation is that the effects of changes in the estimated quantity must be "observable" in the measured quantities.) Furthermore, the incorporated models allow the Kalman filter to estimate not only the state variables of the system, but also the internal parameters of the system under test and the systematic errors of the measurement system. The models incorporated in the Kalman filter are dynamical; that is, they include the dynamic relationships between the system variables. For that reason, those dynamic relationships can be used to improve the estimation process.

The Kalman filter (estimator) can be considered to be a combined simulation of the system under test and the measurement system, in which an automatic controller adjusts the estimates of the system variables and parameters until the results of the simulation agree with the continuing measurements.

Assume that the system under test is represented by the dynamical equation:

$$\dot{\underline{X}} = A(t)\,\underline{X}(t) + B(t)\,\underline{\mu}(t) + G(t)\,\underline{\omega}(t)$$

where $A(t)$, $B(t)$, and $G(t)$ are time-varying matrices, $\underline{X}(t)$ is the state vector of the system, $\underline{\mu}(t)$ is the input vector of the system, and $\underline{\omega}(t)$ is an additive input random noise vector. Assume, also, that the measurement vector, $\underline{3}(t)$,

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is given by the equation:

$$\underline{3}(t) = H(t)\,\underline{x}(t) + \underline{v}(t)$$

where H(t) is a time-varying matrix and $\underline{v}(t)$ is an additive measurement system random error vector. The Kalman filter (estimator) equation is then:

$$\dot{\hat{\underline{x}}} = A(t)\,\hat{\underline{x}}(t) + B(t)\,\underline{u}(t) + K(t)\left[\underline{3}(t) - H(t)\,\hat{\underline{x}}(t)\right]$$

where $\hat{\underline{x}}(t)$ is the estimate of the state vector and K(t) is the Kalman gain matrix. The elements of the Kalman gain matrix are proportional to the cross-sensitivities of the system variables and parameters, proportional to the estimates of the variances of the errors in the estimates, and inversely proportional to the assumed variances of error in the measurements. Thus, the estimation equation constitutes a simulation of the system under test, driven by a term proportional to the differences between the measured and estimated values of the measured quantities. The Kalman gain is computed continually, and ensures a statistically optimal best estimate. It is given by the equation:

$$K(t) = P(t)\,H^T(t)\,R^{-1}(t)$$

where $H^T(t)$ is the transpose of the matrix H(t), $R^{-1}(t)$ is the inverse of the covariance matrix of the measurement errors, $\underline{v}(t)$, and P(t) is the covariance matrix of the (estimated) errors in the estimates. The matrix P(t) is con-tinually computed by integration of the equation:

$$\dot{P}(t) = A(t)\,P(t) + P^T(t)\,A^T(t) - P(t)\,H^T(t)\,R^{-1}(t)\,H(t)\,P(t)$$

$$+ G(t)\,Q(t)\,G^T(t)$$

where Q(t) is the covariance matrix of the system input noise, $\underline{\omega}(t)$.

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As an example of Kalman filtering, consider the scalar sequential estimation of a single system variable, X(t), utilizing continuing measurements, Z(t), where the system model is that of a first-order system, given by the equation:

$$\overset{\circ}{x}(t) = -3\,x(t) + 5t + \omega(t)$$

in which the term 5t represents a deterministic input and ω(t) represents a random noise input with variance σ_ω^2 . Let the measurement system model consist of the equation:

$$z(t) = x(t) + v(t)$$

where v(t) represents additive random measurement error with variance σ_v^2 . The Kalman estimator equations are then:

$$\overset{\circ}{\hat{x}}(t) = -3\,\hat{x}(t) + 5t + (1/\sigma_v^2)\,p(t)\left[z(t) - \hat{x}(t)\right]$$

$$\overset{\circ}{p}(t) = -6\,p(t) - (1/\sigma_v^2)\,p^2(t) + \sigma_\omega^2$$

Note that the Kalman estimator equations are those of a first-order system with a dynamic model identical to that of the system under test, driven by the difference between the measurements, z(t), and the estimate, X(t), weighted by the estimate, p(t), of the variance of the estimation error. The factor $(1/\sigma_v^2)$ weights the driving function inversely to the variance of the measurement error.